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TEACHERS' MANUAL

NUMERICAL ANSWERS

ANSWERS TO SEPARATE
ACHIEVEMENT TESTS

Plane Geometry

WELCHONS
KRICKENBERGER
PEARSON

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BOSTON · NEW YORK · CHICAGO · ATLANTA
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TEACHERS' MANUAL

INTRODUCTION

The suggestions in this Teachers' Manual for Plane Geometry will serve their purpose best if they give you other ideas—your own ideas, adapted to your own classes and to your own ways of teaching. No manual can give a procedure that is exactly right for all classes and all teachers.

To get the most help from the manual, first look through it to see the kinds of help it provides. Notice the first section, "Characteristics of Today's Geometry Teaching." Since the textbook has been planned to conform to the characteristics described in this section, careful reading will help you to understand why the textbook emphasizes particular points and why it includes certain subject matter.

Notice the second section, "Today's Objectives in Geometry Teaching." Whether you use the objectives it lists or a set of your own, you will want to direct your teaching toward particular goals. In this way you will avoid confusion for students and frustration for the teacher.

The next section, "Basic Principles of Geometry Teaching," discusses a number of important topics. Further reading in books on psychology and the teaching of mathematics will help you to keep abreast of the work being done in these fields.

The section "Teaching Aids" brings together for your convenience lists of books, pamphlets, films, and filmstrips to help you make your teaching more effective and more enjoyable. The brief notes following most of the listings will help you to know the areas in which they will be most effective. The films are arranged according to the chapter in which they will probably be most useful to you.

The section "General Suggestions for Using the Text" gives you help in teaching basic concepts, theorems, corollaries, and exercises. There are also suggestions for enriching and deepening your teaching.

There is a "Time Schedule" on page 29 for use by those who want help in planning the number of days to spend on particular topics.

The major part of the manual consists of "Specific Suggestions for Teaching the Various Chapters." Observe that for each chapter there is listed: the objectives of the chapter, a suggested number of days for study of the chapter, suggested content for various courses of study, and specific suggestions for teaching each topic.

CHARACTERISTICS OF TODAY'S GEOMETRY TEACHING

You are teaching geometry in the last half of the twentieth century and your teaching methods and your general outlook should reflect the spirit of these times. In the following paragraphs we have set down some of the characteristics that distinguish today's geometry teaching from the geometry teaching of other years.

Increased Emphasis Upon the Nature of Deductive Reasoning and the Meaning of Proof

Today there is greater emphasis upon helping students to understand the nature of deductive reasoning and the meaning of proof. We discuss with students the role of undefined terms, definitions, and assumptions in deductive reasoning. Before formal geometric proof is begun, we show students how to form simple syllogisms and how to combine syllogisms into longer proofs.

Having started formal geometry, we lead classes to watch the subject grow into a system of thought stemming from the assumptions and undefined terms we are willing to accept and the definitions we are willing to make. As the students progress, we provide opportunities for them to deduce small systems of thought within the larger system.

We lead classes to make comparisons between inductive and deductive methods of reasoning and to note that while the reasoning of science must be in part inductive, the reasoning of mathematics is deductive.

Increased Emphasis Upon Non-Geometric Reasoning

In keeping with the theory that ability to reason deductively in non-geometric situations will follow the study of geometry only provided that students have been taught to apply their knowledge of deduction to non-geometric situations, we introduce many exercises in non-geometric reasoning. We also lead students to analyze common errors in reasoning.

New Freedom in Discussing the Role of Assumptions

Today we feel a new freedom in discussing the existence of other geometries with beginning geometry students. To make the point clear, review a bit of history. Euclid attempted to build a system of absolute geometric truth. By starting with carefully constructed definitions and with axioms and postulates which he felt were true beyond question, he was able to deduce a system of geometric thought which he felt could not be disputed. Not only was his geometry logical, it was consistent with the world which people could see, touch, and move through. It was the only geometry until the first half of the nineteenth century when Nicholas Lobachevsky and John Bolyai questioned his assumption that only one line can be drawn parallel to a given line through a point not on this line (see page 135 of the textbook). Using other assumptions, they were able to build systems of geometry just as logical as Euclid's and, depending upon point of view, also possibly descriptive of the universe. This opening wedge, amplified by our realization that many of the world's greatest scientific and mechanical discoveries have been made by men who have broken with convention to launch out upon new assumptions not consistent with current thought, has gradually brought us to the point of view that Euclid's geometry is only one of many possible geometries; that while Euclid's geometry describes the observed world of here and now, it may not be the best geometry for describing the totality of time and space when more is known about them. In fact, we now feel that geometry (or any other mathematics) does not need to describe the universe; that it can be perfectly good mathematics and still be purely a system of thought.

Today, therefore, we do not tell students that the usual axioms and postulates of Euclidean geometry are self-evident truths. We say that they are assumptions which we are willing to accept, and we point out that in building a system of thought, the assumptions we make affect the conclusions we reach. Not only do we say this, we demonstrate by having classes practice upon exercises in which they draw necessary conclusions from given assumptions. We point out that Euclidean geometry accurately describes the world as we know it, and is consequently useful, because it begins with assumptions consistent with our observations. However, we do not hesitate to state that by changing the assumptions we can form other systems of geometry.

Increased Attention to Integrating Geometry with Other Bodies of Mathematical Knowledge

Another characteristic of today's geometry teaching is our increased emphasis upon the relationships between synthetic geometry and other bodies of mathematical knowledge, particularly arithmetic, algebra, trigonometry, and coordinate geometry. For a number of years a unit on trigonometry has been considered a standard part of a geometry course. Today we also add units on coordinate geometry and on approximate number. The inclusion of coordinate geometry not only helps students to see the interdependencies of algebra and geometry, but also gives them an introduction to a powerful new method of proof and a new way to determine geometric relationships while synthetic geometry is still fresh in their minds. The study of approximate number helps to tie theoretical geometry to the geometry of the practical world of work.

In some schools concepts and theorems of solid geometry are also considered in the plane geometry course. Sometimes this is done purely to help students see the relationships between plane and solid geometry and sometimes it is done because the solid geometry course is broken into parts, some parts being given in plane geometry and some in algebra and other mathematics courses.

Increased Emphasis Upon Producing Mathematical Competency in Students

The present demands for students with more and better mathematics training implies demands for students with both greater ability to reason deductively and more complete understanding of mathematical facts and principles. Today we try to teach for real understanding. We smile over the old-time theory that students can learn geometry by memorizing Euclid's theorems. We even smile over the more modern belief that students can learn definitions and procedures by memorizing them. We realize that there must be some memorization of facts in mathematics, but we feel that memorization must follow understanding, not precede it.

We feel that understanding comes only when students are active participants in exploring and developing meanings and finding their applications. Methods of exploration and development vary according to the concept being explored and developed, and according to the students doing the learning. For example, to lead students to recognize that two triangles are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other, some students will need to draw, cut out, and place one on the other, the triangles of several pairs which meet the requirements of the statement. Other students will grasp the idea quickly through class discussion of the relationships between the parts of a pair of such triangles drawn on the board.

We feel that as soon as understanding is achieved, students should be given practice in using the newly discovered learning. We therefore follow the exploration with exercises which call for use of the new learning. For example, as soon as students have convinced themselves that two triangles are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other, we provide exercises in which pairs of triangles meeting the requirements are to be proved congruent.

Today, we also attempt to lead each student to work to the level of his capacity and to advance as far as he can in mathematics. Obviously, not all students can achieve the same competency, nor progress at the same rate, but now we no longer ask what mathematics a particular student should learn. We know that each individual can find use for all the mathematics he knows, so we encourage each student to proceed as far as he can.

TODAY'S OBJECTIVES IN GEOMETRY TEACHING

The characteristics of today's geometry teaching come to focus in the following objectives

1. To develop in the student an understanding of deductive reasoning. This includes developing an understanding of
    - a. The importance of definitions and the necessity for undefined terms
    - b. The meaning and use of assumptions
    - c. The meaning of "logical sequence"
    - d. The meaning of "necessary and sufficient"
    - e. The meaning of deductive reasoning as contrasted with inductive reasoning and the part each plays in scientific investigation
2. To give the students practice in using deductive reasoning
    - a. In building sequences of theorems and original exercises
    - b. Through applying it to non-geometric situations
    - c. Through analysis of common errors in reasoning
3. To increase the student's store of knowledge of geometric facts, principles, and relationships needed in such fields as engineering and drafting.
4. To help students see the interrelationships between geometry and other areas of mathematics—particularly arithmetic, algebra, trigonometry, and coordinate geometry.
5. To help students appreciate the geometrical nature of the universe, as we know it.

BASIC PRINCIPLES OF GEOMETRY TEACHING

Motivation

Most geometry students have willingly elected the subject and want to succeed in it. With care the teacher can cultivate this interest and keep it flourishing.

One of the surest ways to keep students motivated is to enable them to succeed. Students who fail at least half of the time soon stop trying. On the other hand students who are never challenged by possible failure lose interest through boredom. The wise teacher therefore tries to plan in such a way that students will succeed most of the time, but not every time. He is also liberal with praise for a job well done. Praise draws out the best in us and makes us try harder. Too much criticism leaves us deflated and discouraged and keeps us from doing our best.

Discussions which tie geometry to the interesting things taking place in the world often catch the interest of students and act as motivating forces. Student preparation of reports on particular topics, projects, investigations, trips, bulletin board displays, and specialized reading to open up new geometric vistas is also helpful.

Another stimulus can be provided if students are given opportunities to help direct their own learning activities. This can be done through discussion periods in which the objectives of the course are discussed and planning is done for meeting the various goals. Group planning for special projects also helps. So do differentiated assignments in which students have a measure of choice.

But the greatest motivating force of all is the teacher's own enthusiasm for the subject and his genuine interest in his students.

Individual Differences

All geometry teachers know that students vary in abilities, interests, and needs, and all teachers are concerned as to how to make their teaching reach every member of a class whose members exhibit wide differences.

Ability grouping of students provides a partial answer in schools large enough to make such a plan administratively possible, but it must be realized that even within a comparatively homogeneous class students differ. In schools where ability grouping by classes is impossible, it is sometimes possible to do grouping within the class provided the class period is long enough and other conditions favorable. Differentiated assignments and special exercises for special credit provide other solutions. Enrichment activities and materials for deepening the understanding and widening the geometrical outlook of more able students can also be used to advantage.

In general, slow students need more repetition, more experience with concrete materials, simpler reasoning, and more direction.

It is to meet the needs of students of various ability levels that the exercises of the textbook are divided into three groups—A, B, and C. The A exercises are for all students. They are easy, but since many of them emphasize basic ideas, good students can profit from studying them. The B exercises are of average difficulty. The C exercises are more difficult, yet entirely within the ability of capable high school students.

Help on planning courses for slow, average, and able students is given in the manual under "Specific Suggestions for Teaching the Various Chapters."

Concept Formation

Concept formation seems to originate in sensory experience. To acquire the concept "ball" the child must have many experiences in which he hears the sound "ball" at the time he sees or is handling a ball. Gradually he extracts the meaning "ball" and attaches it to the sound "ball."

It would be extremely difficult for a child who had never seen or handled a triangular shape to form the concept "triangle," though possibly by combining such previously acquired concepts as "three," "line," "intersect," and "distinct" he might be able to do so. Many of the concepts of geometry fall into this classification, and even though they are not entirely based upon sensory experience, they are at least aided by such experience. Other concepts, such as that of "proof," are based almost entirely upon previously acquired concepts, some of them rather far removed from sensory experience.

When it is possible to initiate or extend concept formation by use of objects that students can see and handle, teachers should by all means introduce them, particularly in classes less able to do abstract thinking. As soon as these objects have served their purpose, pictures and diagrams should be substituted to bridge the gap between complete concrete experience and complete abstraction. Finally, the situation can be reduced to words. In cases in which it is impossible to develop a concept through concrete materials (as in the development of the concept of proof) the teacher must rely entirely upon words. In such cases the utmost care must be used to help students relate contributing concepts to the new concept. The words used must have meaning.

Teachers must realize that concept formation takes time. It is only after various experiences in which the concept is involved that complete understanding develops; indeed it is possible that there is not such a thing as complete understanding. It may be that there is only continued growth in understanding. Teachers can aid concept formation by planning recurring experiences, each enough different from past experiences to widen and deepen meanings already developed.

Sensory Experience

The importance of sensory experience can scarcely be overemphasized, yet a few words of warning seem wise. Today teaching gadgets are both plentiful and attractive, and their virtues so well advertised that it is difficult for teachers to keep their sense of values. Many teachers, in their enthusiasm, adopt aids that are of no real value in teaching the concepts which they are supposed to teach. The gadgets become interesting playthings, but the intended learnings do not follow. Other teachers adopt useful multi-sensory aids but overlook the fact that there comes a time when the student must move out beyond material objects into the area of abstract thinking. Many of these teachers would be horrified to find their students counting on their fingers, yet develop similar dependencies upon material objects through overemphasizing gadgets. Geometry should be a course in thinking, not a course in the operation of multi-sensory aids.

The chalkboard is one almost indispensable visual aid. Chalkboard drawings, proofs written on the board, and chalkboard summaries all help students to understand what is being said. Colored chalk can be used to excellent advantage to designate corresponding parts, overlapping triangles, construction lines, loci, and so on. A felt board can be used to advantage to designate overlapping triangles, but has one disadvantage—it leaves the impression that a polygon is the space enclosed by a broken line rather than the line itself. Models of cones, conic sections, cubes, and so on are almost indispensable. Bulletin board displays, collections, posters, films, linkages, pictures, all help to develop understanding. The list is almost endless, but the teacher must use judgment in selection and adaptation.

Language

In addition to being vital in the formation of concepts, language can both reveal faulty concepts and lead to faulty concepts. The students who say, "Angles opposite equal sides are equal," may really believe that in any polygon angles opposite equal sides are equal. The teachers's task in this case is obviously to correct the concept. But it is possible, in fact common, for students to give incorrect statements in spite of correct concepts. Thus, the student who says, "Angles opposite sides are equal," may intend that the statement refer only to triangles. When pressed for an accurate statement, he can produce it, but without the pressure he cuts corners by imitating a familiar word pattern with a minimum of thought. The teacher's task in this case is to insist upon the correct statement. Not only does the incorrect statement leave the wrong impression in the minds of those who hear the statement, but eventually it leads to incorrect thinking on the part of the student who gives it. The student who gives such a statement as, "Angles opposite equal sides are equal," usually begins to believe that in any polygon angles opposite the equal sides are equal.

Problem Solving

Probably no part of geometry is more difficult to teach than problem solving. "Problem" is used here in a general sense to mean any perplexing question demanding settlement, for example, a proof to be planned, a question to be answered, or a construction to be made. We tell students that to solve problems they must relate what they want to find to something they already know. Some students succeed amazingly well; others have difficulties.

Since the more relationships a student recognizes, the easier it is for him to solve problems, frequent summaries of relationships are helpful. For example, during the days a class studies parallelograms, it is helpful to summarize on the chalkboard: "Facts I know about a parallelogram" and "Ways in which I can prove that a quadrilateral is a parallelogram." The lists can grow as more and more relationships are studied. The lists are made more valuable if they are actually used. For example, as one exercise using the lists mentioned above, the class might be asked to prove that the line segments joining the midpoints of the consecutive sides of any quadrilateral form a parallelogram. Before the proof is begun the class should discuss which of the various relationships seems to offer the best possibilities for solving the problem. Classes should be taught to use the summary at the end of each chapter, beginning with Chapter 3. Particular attention should be given to the "Planning the Proof" sections of the proofs given in the textbook, for example, the proof of Theorem 2 on page 102. Students should be encouraged to do similar planning for their own proofs.

Generalizations help students to group families of relationships into single statements. For example, on pages 332 and 333 of the textbook, all the theorems of those pages which relate to a circle and its angles have been gathered into one statement. Such generalizations emphasize relationships.

Encouraging students to search through their texts to find the answers to perplexing questions helps to make them self-reliant in discovering relationships. Teaching proper use of the index also helps.

Use of judicious questions which give students clues without taking away the challenge of a problem helps in problem solving. Ordinarily nothing is gained by allowing students to flounder. This does not mean, however, that able students should never be given difficult exercises with which to wrestle for several days or even several weeks. Usually these able students are not really floundering. Their work is organized and directed even though it may require considerable time for them to find the right combination of relationships.

An intended problem is not really a problem to the student unless it challenges him. Long lists of exercises all done by the same method do not challenge students, at least not after the pattern is learned. While our goal is to get students to the point where a pattern becomes so familiar that it is no longer a problem, nothing is gained by endless repetition beyond the point of familiarity. Of course, students must review and summarize, but even reviews and summaries can be made challenging. Sometimes the mere rewording of an old question, or a new twist to the information given in it, will force new and deeper thinking from students. For example, a class that has been answering such questions as, "What is the sum of the interior angles of a pentagon?" must do a little extra thinking when next day the question is asked as, "The sum of the interior angles of a pentagon is equal to how many right angles?"

Students gain greater insight into geometric relationships by considering several solutions for one problem than by using one solution for several problems. For example, a class gains more from attempting to find several ways of constructing a line parallel to a given line through a given point, than by using a particular way over and over again for various pairs of lines and points.

TEACHING AIDS

Books and Pamphlets

Journals and Yearbooks

Central Association of Science and Mathematics Teachers, Inc. School Science and Mathematics. Monthly except July, August, and September. Box 408, Oak Park, Illinois.

National Council of Teachers of Mathematics. The Mathematics Teacher. Monthly except June, July, August, and September. 1201 Sixteenth Street, N.W., Washington 6, D.C.

National Council of Teachers of Mathematics. Yearbooks. Periodically since 1926. 1201 Sixteenth Street, N.W., Washington 6, D.C. Among those particularly interesting to geometry teachers are: The Nature of Proof, 1938; Multi-Sensory Aids in the Teaching of Mathematics, 1945; The Learning of Mathematics—Its Theory and Practice, 1953; Emerging Practices in Mathematics Education, 1954; Insights into Modern Mathematics, 1957.

Books on Teaching Methods

Boyer, Lee E. Introduction to Mathematics, Revised Edition. New York: Henry Holt and Company, Inc., 1956.

Brown, Claude H. The Teaching of Secondary Mathematics. New York: Harper & Brothers, 1953.

Butler, C. H. and Wren, F. L. The Teaching of Secondary Mathematics, Second Edition. New York: McGraw-Hill Book Company, Inc., 1951.

Commission on Secondary School Curriculum, Progressive Education Association, New York (Committee on the Function of Mathematics in General Education). Mathematics in General Education. New York: Appleton-Century-Crofts, Inc., 1940.

Fehr, Howard F. Secondary Mathematics: A Functional Approach for Teachers. Boston: D. C. Heath and Company, 1951.

Kinney, Lucien B. and Purdy, C. Richard. Teaching Mathematics in the Secondary School. New York: Rinehart and Company, Inc., 1952.

O'Brien, Katharine E. Successful Devices in Teaching Geometry. Portland, Maine: J. Weston Walch, 1954.

Reeve, Wm. David. Mathematics for the Secondary Schools—Its Content and Methods of Teaching and Learning. New York: Henry Holt and Company, 1954.

Walch, J. Weston and Cordell, Christobel M. Colorful Teaching of Mathematics. Portland, Maine: J. Weston Walch, 1955.

Books for General Understanding of Mathematics

Abbott, Edwin A. Flatland. New York: Dover Publications, Inc., 1952. An imaginative description of life in a two-dimensional world.

Altshiller-Court Nathan. College Geometry. Richmond: Johnson Publishing Company, 1925. An advanced book on synthetic geometry containing many exercises within the range of high school ability.

Bakst, Aaron. Mathematics, Its Magic and Mastery, Second Edition. Princeton, New Jersey: D. Van Nostrand Company, Inc., 1952. Discusses the versatility of mathematics.

Ball, Walter W. R. Mathematical Recreations and Essays, revised by H. S. M. Coxeter. New York: The Macmillan Company, 1953. A revision of an old book of recreational mathematics.

Beeler, Nelson F. and Branley, Franklyn, M. Experiments in Optical Illusion. New York: Thomas Y. Crowell, 1951.

Bell, Eric T. Mathematics, Queen and Servant of Science. New York: McGraw Hill Book Company, Inc., 1951. Suitable for high school seniors who want to understand how mathematics serves science.

Bell, Eric T. Men of Mathematics. New York: Simon and Schuster, 1937. Contains biographies and comments on the work of thirty great mathematicians.

Cooley, Hollis R., and others. Introduction to Mathematics. New York: Houghton Mifflin Company, 1949. Suitable for a college survey course. Good reference book for teachers.

Courant, R. and Robbins, H. What is Mathematics? New York: Oxford University Press, Inc., 1941. A comprehensive survey of mathematics which leads students from elementary to advanced considerations of each topic treated.

Eves, Howard. An Introduction to the History of Mathematics. New York: Rinehart and Company, Inc., 1953. Excellent material for special projects and problems.

Gamow, G. One, Two, Three---Infinity. New York: The Viking Press, 1947. A collection of interesting facts and theories of modern science.

Kasner, E. and Newman, J. Mathematics and the Imagination. New York: Simon and Schuster, 1940. A collection of fascinating facts about mathematics.

Kramer, Edna E. The Main Stream of Mathematics. New York: Oxford University Press, Inc., 1951. A fascinating history of mathematics.

Lieber, L. and Lieber, H. Non-Euclidean Geometry. New York: Galois Institute, 1940. A delightful book that makes the elements of non-Euclidean geometry understandable.

Lieber, L. and Lieber, H. The Einstein Theory of Relativity. New York: Rinehart and Company, Inc., 1945. Light-hearted introduction to Einstein theory of relativity.

Lieber, L. and Lieber, H. The Education of T. C. Mits. New York: W. W. Norton and Company, Inc., 1944. Stresses implications of the postulational method in daily life. Whimsical, entertaining, and sound.

Lieber, L. and Lieber, H. Mits, Wits, and Logic. New York: Galois Institute, 1947. Entertaining presentation of the basic elements of formal logic.

Newman, James R. (editor). The World of Mathematics (4 vols.). New York: Simon and Schuster, 1956. A compendium of all aspects of mathematics.

Ransom, William R. One Hundred Mathematical Curiosities. Portland, Maine: J. Weston Walch, 1955.

Sawyer, W. W. *Mathematician's Delight*. Baltimore: Penguin Books, Inc., 1943. Written to convince the general reader that mathematics is not a forbidding science.

Sawyer, W. W. *Prelude to Mathematics*. Baltimore: Penguin Books, Inc., 1955. Contain accounts of some interesting, stimulating, and surprising branches of mathematics.

Steinhaus, Hugo. *Mathematical Snapshots*. New York: Oxford University Press, Inc., 1950. Presents some of the more unusual aspects of mathematics, including mathematical paradoxes.

Wolfe, Harold E. *Introduction to Non-Euclidean Geometry*. New York: The Dryden Press, 1945. First chapters present the foundations and history of the subject. Good for class reports.

Yates, Robert C. *Geometrical Tools*. St. Louis: Educational Publishers, Inc., 1949. Contains much enrichment material on folds and creases, linkages, and constructions with restricted means.

Yates, Robert C. *A Handbook of Curves and Their Properties*. Ann Arbor: J. W. Edwards, Publisher, Inc., 1952. A teacher's handbook.

Yates, Robert C. *The Trisection Problem*. Ann Arbor: J. W. Edwards, Publisher, Inc., 1947. A brief history and proof of the impossibility of trisecting an angle.

Book of Review and Supplementary Exercises

Smith, David P., Jr. and Fagan, Leslie T. *Mathematics Review Exercises*. Boston: Ginn and Company, 1956. An excellent problem review book. Contains problems from arithmetic, algebra, plane geometry, solid geometry, and trigonometry.

Bulletins and Pamphlets

Association for Supervision and Curriculum Development. *What Does Research Say About Secondary Mathematics*? ASCD a Department of NEA, Washington, D.C., 1955.

Brown, K. E. and Johnson, P. G. *Education for the Talented in Mathematics*. A Report of a Joint Conference of the Cooperative Committee on the Teaching of Science and Mathematics of the American Association for the Advancement of Science and the United States Office of Education. Washington: Supt. Of Documents, 1953.

Johnson, Donovan A. *Paper Folding for the Mathematics Class*. Washington: National Council of Teachers of Mathematics, 1957.

National Association of Secondary School Principals. *Mathematics in Secondary Schools Today*. Washington, 1954.

National Council of Teachers of Mathematics. *Guidance Pamphlet in Mathematics for High School Students*, Revised Edition. Washington, 1953.

National Council of Teachers of Mathematics. *How to Do It Series*: *How to Use Your Bulletin Board*, *How to Use Field Trips in Mathematics*, *How to Develop a Teaching Guide in Mathematics*, *How to Use Films and Filmstrips*. Washington, 1955-1956.

Swain, Henry. How to Study Mathematics: A Handbook for High School Students. Washington: The National Council of Teachers of Mathematics, 1955.

Student Journals

Mathematics Student Journal, National Council of Teachers of Mathematics.

Mathematics Student, Brooklyn Technical High School.

O. U. Mathematics Letter, University of Oklahoma.

Films and Filmstrips

Introduction to Geometry

Films:

Geometry And You: 1 reel color or b&w (Coronet)
Demonstrates the everyday importance of the function of geometry.

Filmstrips:

Introduction To Plane Geometry: color (CurriclmFlms)

Geometric Figures: b&w (McGrawHill)

Introduction To Plane Geometry: b&w (SVE)

Chapter 1—Lines and Angles

Films:

Lines and Angles: 1 reel b&w (KnowledgeBldrs)
Illustrates the relationship of the perpendicular with the ordinary plumb bob, level and square.

Angles: 1 reel b&w (KnowledgeBldrs)
Helps clarify angles by means of intersecting lines and measurement with a protractor.

Congruent Figures: 1 reel b&w (KnowledgeBldrs)
Demonstrates the geometric principles for finding and proving that triangles with "equal sides", "equal angles", or the combination of both, are equal and congruent.

Filmstrips:

Vocabulary: Lines And Angles I: color (CurriclmFlms)

Vocabulary: Lines And Angles II—Relationships: color (CurriclmFlms)

Vocabulary: Lines, Relationships—Direction, Perpendicular Lines: color (CurriclmFlms)

Addition and Subtraction In Geometry: b&w (JamHandy)

Measurement: b&w (McGrawHill)

Basic Angles And Experimental Geometry: b&w (SVE)

Chapter 2—Inductive Reasoning

Films:

How To Find The Answer (Mathematical Problem Solving): 1 reel color or b&w (Coronet)
Emphasizes clear thinking in demonstrating a systematic attack on mathematical problems.

Using The Scientific Method: 1 reel color or b&w (Coronet)
Presents the scientific method in the context of an everyday problem.

The Scientific Method: 1 reel color or b&w (EBF)
Explains the steps in the scientific method and demonstrates the value of scientific thinking in dealing with problems of everyday life.

Filmstrip:

Applying Geometric Logic—Induction, Analysis, and Indirect Reasoning: color (CurriclmFlms)

Chapter 3—Deductive Reasoning

Filmstrips:

Foundations Of Geometry—Postulates: Lines: color (Curriclm Flms)

Applying Geometric Logic—Definitions And Key Words: color (CurriclmFlms)

Applying Geometric Logic—Deductive Reasoning: color (CurriclmFlms)

Applying Geometric Logic—Mistakes In Thinking: color (CurriclmFlms)

Multiplication And Division In Geometry: b&w (JamHandy)

Introduction To Demonstrative Geometry: b&w (SVE)

Chapter 4—Triangles

Films:

Similar Triangles: 1 reel b & w (KnowledgeBldrs)
Shows properties of similar triangles and demonstrates the "two angles equal" theorem.

Properties Of Triangles: 1 reel b&w (KnowledgeBldrs)
The practical importance of the rigidity of triangles is illustrated by comparing the strength of triangles with the strength of non-rigid figures.

Filmstrips:

Vocabulary: Triangles: color (CurriclmFlms)

Basic Triangles: b&w (SVE)

Congruent And Overlapping Triangles: b&w (SVE)

Chapter 5—Parallel and Perpendicular Lines

Films:

Parallel: 1 reel color or b&w (JohnsonHunt)
Develops an understanding of the concept and laws of parallel lines and transverse angle relationships.

Filmstrips:

Vectors: b&w (JamHandy)

Parallel Lines And Transversals: b&w (SVE)

Chapter 6—Constructions

Filmstrips:

Constructions: b&w (JamHandy)

Chapter 7—Polygons

Films:

Polygons: 1 reel b&w (KnowledgeBldrs)
The definition of polygon and the ideas of convex and concave are clarified by animation.

Quadrilaterals: 1 reel b&w (KnowledgeBldrs)
Illustrates and explains the chief properties of the important quadrilaterals, including parallelograms, rectangle, rhombus, square, trapezoid and trapezium.

Filmstrips:

Vocabulary: Polygons: color (CurriclmFlms)

Quadrilaterals: b&w (SVE)

Similar Polygons: b&w (SVE)

Chapter 8—Areas of Polygons

Films:

Areas: 1 reel b&w (KnowledgeBldrs)
Graphic demonstrations of methods for computing the area of rectangles, parallelograms, triangles, and circles.

Pythagorean Theorem: 1 reel b&w (KnowledgeBldrs)
Proves the theorem and demonstrates with examples.

Filmstrips:

Areas: b&w (SVE)

Chapter 9—Circles, Angles, and Arcs

Films:

The Circle: 1 reel b&w (KnowledgeBldrs)

Presents and clarifies radii, diameters, chords, tangents, secants, arcs and central angles.

Chords And Tangents Of Circles: 1 reel b w (KnowledgeBldrs)
Deals with the theorem on a perpendicular to a chord within the circle.

Filmstrips:

Vocabulary—Circles I: color (CurriclmFlms)

Vocabulary—Circles II: color (CurriclmFlms)

Geometry In Art: color (CurriclmFlms)

Introduction To Circles: b&w (SVE)

Common Tangents And Tangent Circles: b&w (SVE)

Chapter 10—Measurement of Angles and Arcs

Films:

Angles And Arcs In Circles: 1 reel b&w (KnowledgeBldrs)
Portrays the measurement of central angles, arcs, inscribed angles and angles formed by two chords.

Indirect Measurement: 1 reel b&w (KnowledgeBldrs)
Indirect measurement techniques are established by showing specific situations where direct measurements can not be made.

Filmstrips:

Measurement Of Angles And Arcs: b&w (JamHandy)

Indirect Measurement: b&w (McGrawHill)

Chapter 11—Loci

Films:

Locus: 1 reel b&w (KnowledgeBldrs)
The concept of locus is visualized and explained by a combination of animated drawings, photographic motion and the spoken word.

Rectangular Coordinates: 1 reel b&w (NuArt)
A graphic presentation of Descartes' concept.

Filmstrips:

Locus: color (CurriclmFlms)

Loci: b&w (SVE)

Chapter 12—Proportion and Proportional Line Segments

Films:

Ratio and Proportion: 1 reel b&w (KnowledgeBldrs)
The relationship between a ratio and a proportion is clarified.

Filmstrips:

Ratio and Proportion: b&w (JamHandy)

Chapter 13—Similar Polygons

Filmstrips:

Scales And Models: b&w (JamHandy)

Chapter 14—Regular Polygons and the Circle

Films:

The Meaning Of Pi: 1 reel color or b&w (Coronet)
Outlines a study procedure to check and review the numerical value of pi.

Filmstrips:

Variables And Coordinates: b&w (McGrawHill)

Chapter 15—Elements of Trigonometry

Filmstrips:

Trigonometry: b&w (JamHandy)

Chapter 16—Coordinate Geometry

Filmstrips:

Analytic Geometry: b&w (JamHandy)

Chapter 17—Additional Topics

1. Aeronautics

Films:

Theory Of Flight: 1 reel b&w (EBF)
Explains the basic principles of airplane flight by illustrating the application of physical laws to forces acting on airfoils.

Problems Of Flight: 1 reel b&w (EBF)
Animated drawings superimposed over natural photography clarify explanations of force acting upon a plane during flight.

Aerodynamics—Air Flow: 1-1/4 reels b&w (UnitedWorld)
Explains turbulence and skin friction through the visualization of smoke flowing over solids of different shapes and forms.

Filmstrips:

Mathematics In Aviation—The Compass: b&w (McGrawHill)

Mathematics In Aviation—Wind Drift: b&w (McGrawHill)

2. Geometric Relations; Orthographic Projection

Films:

Orthographic Projection: 1-1/4 reels b&w (McGrawHill)
For technical training in engineering drawing.

LEGEND TO PRODUCERS AND DISTRIBUTORS

Coronet	Coronet Films, Coronet Building, Chicago 1
CurriclmFlms	Curriculum Films Educational Films, Inc., Distributor 18 East 41st Street, New York 17
EBF	Encyclopaedia Britannica Films, Inc. 1150 Wilmette Avenue, Wilmette, Illinois
JamHandy	The Jam Handy Organization 2821 East Grand Boulevard, Detroit 11, Michigan
JohnsonHunt	Johnson Hunt Productions 6509 De Longpre Avenue, Hollywood 28, California
KnowledgeBldrs	Knowledge Builders, 625 Madison Ave., New York 22
McGrawHill	McGraw-Hill Book Co., Text-Film Department 330 West 42nd Street, New York 36
NuArt	Nu-Art Films, Inc., 112 West 48th St., New York 19
SVE	Society for Visual Education, Inc. 1345 West Diversey Parkway, Chicago 1, Illinois
UnitedWorld	United World Films, Inc., 1445 Park Ave., New York 29

GENERAL SUGGESTIONS FOR USING THE TEXT

Special Features of the Text

Before you begin to use the text, study its organization and special features.

It is divided into eighteen chapters, each chapter beginning with an introductory statement that describes the nature of the learning to be gained from the chapter. Chapters end with a list of exercises, a word list, and one or two tests. Beginning with Chapter 3, each chapter contains a summary of the principal bases for proof developed within the chapter. A complete summary of the bases for proof is given at the end of the textbook. Tables of measure, lists of formulas, and tables of square roots and of trigonometric ratios are also given at the back of the book, while lists of standard symbols and abbreviations are given at the front of the book.

Each topic of the book is numbered and headed by a key phrase in heavy type. The numbering makes it easy to refer to particular topics, and the topic headings make it easy to locate subject matter. Observe that both definitions and descriptions of undefined terms are underlined in red.

The first three chapters are devoted to developing the basic concepts of geometry, while Chapters 4-14 and 16 develop synthetic geometry as a logical system of thought. Chapters 15, 17, and 18 present the elements of trigonometry, coordinate geometry, and additional topics on aeronautics, geometric relations, and orthographic projection. Chapters 5, 7, 8, 9, 11, 13, and 14 contain selected topics from space geometry. Footnotes throughout the book (for example, on page 111) suggest possibilities for sequence of topics other than the sequences given in the book.

Teaching Basic Concepts

It was stated at the beginning of this manual that one of the characteristics of today's geometry teaching is greater emphasis upon the nature of deductive reasoning and the meaning of proof. In keeping with this theory the text devotes the first three chapters to developing the concepts necessary for such reasoning in geometry.

Chapter 1 is designed to lead students to see the necessity for undefined terms and the importance of definitions. It describes such undefined terms as point and line, and formulates basic definitions for such terms as angle, perpendicular, and complement. It shows the student efficient methods for naming and reading the names of the basic configurations discussed in the chapter.

Chapter 2 explains the nature of inductive reasoning and points out its strengths and weaknesses.

Chapter 3 explains the nature of deductive reasoning and shows students how to build simple syllogisms and how to combine syllogisms to form longer proofs.

It is important to teach the first chapters with the utmost care, for in these chapters a student is given the basic idea upon which he must build his whole system of geometric thought.

Discuss each new topic in class and illustrate it with devices and drawings until you are sure that students have formed adequate concepts. Throughout the course continue to expand and develop these concepts. Many of the exercises of these chapters (particularly the first) are very easy, but should not be omitted. The best learning comes when students follow exploration of concepts with practice in using them. The exercises have purposely been made easy in order to build self-confidence in students.

Do not move into Chapter 4 until you are sure that students have an adequate idea of the meaning of "proof," and know how to proceed in combining a specific statement and a general statement to form a conclusion, as shown in Chapter 3. Move carefully through the first pages of Chapter 4 for these pages give students their first experience in proving triangles congruent.

Teaching Theorems

The theorems of geometry form the main branches of the system of geometric thought and give it structure. In general, teachers find it advisable to consider theorems in class and to use exercises for assignments, but regardless of the procedure you use, make each theorem the subject of class discussion. In the discussion emphasize the theorem as the parent statement for several exercises. Students are thus helped to recognize geometry as an organized system of thought.

Introducing a theorem in class helps the teacher to lead the students to "discover" its proof, whereas when theorems are assigned as homework, students tend to depend upon the proofs, or parts of proofs, given in the text. Scientific investigation indicates that the best learning takes place when students are active participants in discovering concepts and procedures. Lead students to depend upon their own powers of reasoning. (Texts must include many proofs for the benefit of those who study geometry upon an individual basis, and for ease in review, but this does not mean that students under the supervision of a teacher should rely entirely on the textbook version.)

Proofs of theorems are usually developed through a series of exercises preceding the theorem. For example, Exercise 8 of page 137 prepares students for Theorem 9 of page 142. Use of these exercises in homework assignments can force students to discover the proof for a theorem on occasions when you desire to make the proof of a theorem a part of an assignment.

In the remaining pages of this manual frequent reference is made to informal proof as contrasted to formal. A formal proof is a proof written in formal style as explained on page 69 of the textbook. Today we feel that it is not necessary for classes to prove every theorem formally. This does not mean that theorems proved informally are proved less carefully. It simply means that the formal "Theorem, Figure, Given, To prove, Proof" pattern need not be followed exactly.

Informal proof may be handled in a variety of ways depending upon the nature of the proof and the needs of the class. For example, to prove a theorem informally the class may discuss the meaning of the theorem without actually writing the statement. The figure may be placed on the chalkboard and the equal parts marked with tic marks instead of being listed formally in a section headed "Given." The "To prove" may be written on the board to help the class hold it in mind. The teacher may then write the statements of the proof as the class supplies them orally. Reasons may either be written or given orally.

In later pages of this manual, under the specific suggestions for teaching the text, there are sections "Planning the Content of Various Courses." These sections suggest possible divisions of theorems on the basis of need for formal or informal proof. In general, those listed for formal proof are the ones most likely to appear on college entrance and other examinations, but you should use your own judgment when making divisions for your class. The suggestions of the manual simply show one possible way to make a selection.

In formal proofs students should be required to write complete demonstrations as outlined on page 69 of the textbook. Reasons should be written as complete and accurate statements or as standard abbreviations. Teachers must use their own judgments as to what constitutes a complete proof. Mediocre students often become hopelessly confused when too much rigor is required. The important thing is that a student include the principal statements in his argument. In this connection, it should be stated that the proofs given in the textbook are not necessarily to be considered complete. In most cases only the principal statements have been included, and in many cases even these have been cut to a minimum so that the book would not be over-long.

In general, corollaries should be handled in much the same way that theorems are handled.

Teaching Exercises

Since the number of exercises in the text is so large that no student can be expected to do all of them, direct the attention of your students to those that you think will be most helpful. As was explained earlier, the exercises of the text have been divided into A (easy), B (average), and C (difficult) groups. There are many ways of assigning exercises to fit the needs of the various members of your class. One way is to make a minimum assignment of A exercises to be done by all students and to append a list of B and C exercises to be done by students who wish to work beyond the minimum. For example, in assigning exercises from pages 137-138 you might include Exercises 1, 2, 4, 6, 7, and 8 in the minimum group and then list Exercises 12, 13, 14, 16, and 17 as extras to be done by students interested in meeting higher standards. It is not intended that all the exercises mentioned are to be part of a single day's assignment.

In making an assignment of exercises, it is wise to have the class discuss possible procedures for some of the exercises assigned. It is wise, also, to call attention to points that are likely to cause difficulty.

Teach your class how to attack problems by listing all the relationships which seem to offer possibilities for solution, and then to test those relationships which seem most likely to produce the desired result. Teach students to analyze problem situations as shown on pages 156, 157 of the text. Make full use of the aids provided by the textbook. Among such aids are illustrative examples similar to those on page 88, developmental exercises such as those on page 85, and the explanations in §§67, 70, 81, 102, 137, 138, 175, 201, 339, 375, and 408. Teach students to use the chapter summaries and the index.

You can give further help by reminding students that the text is developing a logical system of geometric thought and is helping to add to this system by so arranging exercises that most of them follow directly from the theorem and corollaries immediately preceding. Students are thus building small systems of thought within the larger system. You will need to warn your class to be on guard, however, since occasionally an exercise not belonging to that particular small system of thought is included in a list. Miscellaneous lists of exercises will consist of exercises stemming from many different parent statements.

Testing

Each chapter ends with one or more tests on the content of the chapter. These tests may be used in various ways. Although you may use them to diagnose student difficulties or to test student achievement, we believe that since all students have access to them, it will be better to use them either as practice tests or as regular homework assignments in which students test themselves. You may use them as guides in making other tests for the class. For those who wish to use them, a booklet of Achievement Tests has been prepared for use with the text. These may be ordered from Ginn and Company at a minimum cost.

Enrichment Activities

Use enrichment activities and materials to stimulate interest, deepen understanding, extend learning, and integrate geometry with other areas of learning. Direct class attention to enrichment topics that students may find helpful, but be willing to let student questions and comments lead the class into unexpected activities, provided these activities give promise of being worthwhile. Choose activities that will provide real learning experiences. Use activities that involve individuals as well as those that involve the whole class or a group within the class.

Enrichment may take place through discussion, reading, research, bulletin board displays, field trips, exhibits, study of special topics, science-mathematics fairs, speeches and reports, watching films, looking at pictures other than those on film, making models, making collections, and so on.

Supplementary topics might be some of the following, or others that fit the needs of your particular class: Hexagons in Nature, Circles in Nature, The Golden Section, Symmetry, Other Geometries, The Fourth Dimension, Paradoxes of Geometry, Conic Sections, Topology, The Moebius Strip, Linkages, Great Men of Mathematics, History of Mathematics, Geometry and Architecture, Geometry in Art, and Harmonic Division and Music.

TIME SCHEDULE

The following time schedule has been included for teachers who wish to use it. The number of days listed are given as suggestions only and should be adjusted to meet the needs of particular schools.

	Minimum number of days	Average number of days	Maximum number of days
Chapter 1. Lines and Angles	6	7	8
Chapter 2. Inductive Reasoning	3	3	4
Chapter 3. Deductive Reasoning	9	10	11
Chapter 4. Triangles	16	18	20
Chapter 5. Parallel and Perpendicular Lines	16	18	20
Chapter 6. Constructions	4	4	5
Chapter 7. Polygons	12	14	16
Chapter 8. Areas of Polygons	14	15	16
Chapter 9. Circles, Angles, Arcs	7	8	9
Chapter 10. Measurement of Angles and Arcs	5	6	7
Chapter 11. Loci	8	11	13
Chapter 12. Proportion and Proportional Line Segments	6	7	8
Chapter 13. Similar Polygons	12	14	16
Chapter 14. Regular Polygons and the Circle	6	8	10
Chapter 15. Elements of Trigonometry	4	4	4
Chapter 16. Inequalities	5	6	6
Chapter 17. Coordinate Geometry	7	7	7
Chapter 18. Additional Topics	___	___	___
Totals	140 days	160 days	180 days

SPECIFIC SUGGESTIONS FOR TEACHING THE VARIOUS CHAPTERS

Chapter 1. Lines and Angles

Objectives:

To introduce such basic concepts as point; line, and angle.

To show the place of undefined terms and definitions in the logical study of geometry.

To introduce the mechanics of naming and reading lines and angles, adding and subtracting angles, adding and subtracting line segments, and so on.

Time required: 6-8 days.

Planning the content of various courses:

Basic course—Students should study all topics included in this chapter. All are basic.

Maximum course for students who plan to specialize in mathematics, science, engineering, and similar fields—Same as basic course.

Minimum course—Same as basic course.

Specific Suggestions for Teaching the Various Topics:

Move slowly at the beginning of the course. Students are facing a new area of learning and need time to assimilate new ideas. Students cannot study later chapters successfully if they have failed to grasp concepts developed in the first four chapters.

You may want to start the first class session by having the students read together and discuss page 4 and the first half of page 5. Emphasize the key ideas: (1) Students have already accumulated many ideas concerning size and shape which they are now ready to organize into a logical system of thought. (2) Logical thinking is thinking which correctly combines ideas so as to produce other ideas. (3) The formal study of geometry not only provides important new geometric information, but also provides means for helping us to learn to do better thinking. Emphasize that geometry is not just for engineers, scientists, and mathematicians, but for all people. Everyone needs to be able to think logically. You may want to establish the latter point with illustrations such as the story that Lincoln as a young lawyer bought a geometry book and studied the subject to improve his ability to think logically.

Sec. 1 (p. 5). Importance of Definitions—To read geometry successfully we must proceed carefully, weighing meanings as we read. From the beginning teach students to weigh meanings by having the class read a paragraph or two and then discuss what has been read. Skillful questioning on your part can direct attention to key ideas.

Sec. 2 (p. 5). Undefined Terms—The concept that some terms must remain undefined will be new to most students. It may require time for all members of the class to assimilate the idea.

Emphasize that in this course points are named with capital letters and lines with lower-case letters, though lines may be named by naming points on the line. Be alert to discover students who do not make this distinction in lettering. Lack of conformity on this point leads to confusion later.

Your first assignment might consist of the exercises on pages 8 and 9. Point out that while the exercises are very simple they emphasize basic geometric ideas and should be done carefully.

Secs. 3-4 (p. 9). Definitions, Using Definitions--These paragraphs will answer students later when they object that a definition given in the text is not the definition they have always used. While more than one definition is sometimes possible, the book is based upon the definition as stated. Changing the definition would necessitate changing many statements which depend upon it. While experienced geometry students might be able to make all such changes, in a first course it is better to accept the book definitions.

Secs. 5-6 (p. 12). Distance Between Two Points, Equal Line Segments—Students sometimes find it difficult to understand that line segments are equal if they can be made to coincide. Use rulers of equal length to show that when one ruler is placed along the other with one pair of ends coinciding, the other pair of ends must coincide. Use rulers of unequal length to show that unequal line segments cannot be made to coincide.

Sec. 7 (p. 12). Measurement is Approximate—You may find students who have always assumed that if they work carefully enough with very precise measuring sticks, they can make exact measurements. Discuss some of the reasons why this is impossible—inability to make a perfect ruler; changes in length due to heat and cold, dampness and dryness; inability to set the ruler exactly where we intend to place it; and inability to read it exactly. Use an enlarged ruler or a large picture of a ruler to show what is meant by making a measurement to the nearest tenth of an inch, the nearest hundredth of an inch, and so on.

Sec. 8 (p. 14). How to Add and Subtract Line Segments—Exercises 4-6, 8-12 on pages 15 and 16 help to emphasize the addition and subtraction of line segments. You may need to supplement the book with exercises of your own.

Sec. 9 (p. 15). Constructions in Geometry—Emphasize not only the making of the construction, but also the fact that we do not claim that the construction is exact. Explain that when we say that a construction is correct we only mean that if we could do perfectly what we say we have done, the construction would be perfect. This is usually a new thought to students and only gradually takes on meaning in their minds.

Sec. 10 (p. 16). Angles—Point out that in using three letters to name an angle, the vertex letter is always the middle letter, that is, we read along one side to the vertex and out again along the other side. We never jump the gap in the angle. Stress this method with plenty of practice since students who cannot read angles properly never know what the class is talking about. Watch for the student who uses lower-case letters for points and capital letters for angles.

Sec. 11 (p. 16). Size of Angles—Use two rulers (held to form an angle) to illustrate how the revolution of one side about the other increases and decreases the size of the angle. Form equal angles with two pairs of rulers and show what is meant by the statement, "Two angles are equal if they can be made to coincide." Form one angle with two yardsticks and another with two shorter rulers to show that angles can coincide (be equal) even though their sides are of different lengths. Draw an angle on the board, shorten its sides by erasing the outer portions of each side, then ask whether shortening the sides changed the size of the angle.

Sec. 12 (p. 17). Adding and Subtracting Angles—Exercises such as Exercises 9-13 on page 18 and Exercises 20-22 on page 20 give practice in adding and subtracting angles.

Secs. 13-15 (p. 20). Adjacent Angles, Perpendiculars and Right Angles, Distance from a Point to a Line—Use illustrations, exercises, and discussion to develop meanings. Insist that when students use these definitions, they give the statements completely and accurately. Carelessness in making statements soon leads to careless thinking. Pay particular attention to the definitions for "perpendicular" and for "right angle." Students often fail to notice that the definitions are separate definitions.

Sec. 16 (p. 21). Angles Classified as to Size—Students should become familiar enough with the names of the various kinds of angle to use the names correctly and without hesitation. Do not neglect the exercises. They fix names and concepts.

Secs. 17-18 (p. 22). Measurement of Angles, How to Use the Protractor—Even though most students have studied degrees, minutes, and seconds before they begin formal geometry, many are only vaguely aware of the meanings of the words.

Although the protractor is a measuring device and therefore not generally used in geometry, it is described here because some of the exercises in Chapter 2 require use of a protractor.

Secs. 19-23 (p. 24). Vertical Angles, Complementary Angles, Supplementary Angles, Bisectors, Circle—Ability to use such words as "vertical," "complementary," and "supplementary" at the proper time probably depends mostly on memory, but the concepts involved should not be neglected.

Chapter 2. Inductive Reasoning

Objectives:

To stress the importance of inductive reasoning in discovering laws, principles, and relationships.

To show the inadequacy of inductive reasoning in producing proof.

To point out to students that most of their geometric thought to date has been inductive.

Time required: 3-4 days.

Planning the content of various courses:

Basic course—§§24-28. Students should do all the A exercises and most of the B exercises on pages 33-35, carefully answering all the questions asked. They should also do all exercises on pages 37-38.

Maximum course for students who plan to specialize in mathematics, science, engineering, and similar fields—Basic course plus §29.

Minimum course—Same as basic course.

Specific suggestions for teaching the various topics:

Sec. 24 (p. 31). Discovering Information by Observation—Note the key idea: Even though observation is indispensable, it can lead us into false conclusions.

Sec. 25 (p. 32). Discovering Information Through Measurement—Be prepared to explain that the zero in 1.0" is written to indicate that the measurement has been made to the nearest tenth of an inch.

Sec. 26 (p. 32). Discovering Information Through Experimentation—The relationships discovered in the exercises on pages 33-35 are not important at this time, but it is important for the students to (1) perform the experiments carefully, (2) draw a tentative conclusion from each experiment, and (3) recognize the possibility that the conclusion is incorrect.

By doing one or two exercises at the board with the cooperative efforts of the class you can emphasize a good pattern of procedure for the other exercises. Do not make the assignment so long that students cannot prepare it carefully. A few exercises thoroughly done (with all measurements carefully made and all questions answered) will produce better understanding than many exercises carelessly done.

Sec. 27 (p. 35). Inductive Reasoning—Do not neglect this section. It explains the importance as well as the shortcomings of inductive reasoning.

Sec. 28 (p. 38). Need for Proof—This section links Chapter 2 to Chapter 3.

Sec. 29 (p. 38). Growth of Geometry—This section is not essential to the sequence of topics. It may be studied at any time or may be the basis for a special report by a member of the class.

Chapter 3. Deductive Reasoning

Objective: To teach the basic principles of deductive reasoning.

Time required: 9-11 days.

Planning the content of various courses:

Basic course—All topics of this chapter are basic. By the end of the chapter students should be able to recognize the part played by the general statement, the specific statement, and the conclusion in a deduction, and should be able to identify these in simple proofs consisting of two or three cycles of deduction; should know that general statements may be undefined terms, definitions, or assumptions; and should be able to supply the missing part of a simple deduction in which only two parts are given. Theorem 1 should be proved in class.

Maximum course for students who plan to specialize in mathematics, science, engineering, and similar fields—Same as basic course.

Minimum course—Same as basic course.

Specific suggestions for teaching the various topics:

Teach this chapter with the utmost care. It is basic to the remainder of the course.

Sec. 30 (p. 45). What Deductive Reasoning Is—Make sure that students know what is meant by "general statement" and "specific statement."

Sec. 31 (p. 45). How to Reason Deductively—Make sure that your students understand that every deduction consists of the three parts listed in red at the bottom of page 45. Make sure, too, that they recognize that reasoning is valid only when the specific statement meets all the requirements of the general statement and the conclusion is the one called for in the general statement. § 32 will help students determine whether reasoning is valid.

Sec. 32 (p. 48). The Hypothesis and Conclusion—Students usually have little difficulty in deciding upon the hypothesis and conclusion of a general statement such as those on page 51, but they sometimes have trouble in seeing that the specific statement and conclusion of a cycle of deductive reasoning parallel the hypothesis and conclusion of the general statement of the cycle. You may find it helpful to use a scheme such as the following to indicate the relationship. Using Example 1 at the top of page 46 you might write on the chalkboard:

Specific statement: Martha is a student who expects to graduate from our high school.	General statement—Hypothesis: If students intend to graduate from our high school,
Conclusion: She must take three years of English.	Conclusion: they must take three years of English.

You can now point out that the specific statement expresses the same thought as the hypothesis of the general statement except that it replaces the general word "students" with the specific word "Martha." Likewise, the conclusion of the whole cycle of reasoning expresses the same thought as the conclusion of the general statement except that it replaces the general word "they" with the specific word "she." You may point out that the specific statement and conclusion which follows it are really a specific hypothesis and conclusion, while the hypothesis and conclusion of the general statement are a general hypothesis and conclusion.

The arrangement just described is helpful in determining whether a specific statement satisfies all the conditions of the general statement and whether the conclusion of a cycle of reasoning is one called for in the general statement. In the case just described the specific statement satisfies all the conditions of the general statement because the specific statement expresses the same thought as the hypothesis of the general statement, and the conclusion of the cycle of reasoning expresses the same thought as the conclusion of the general statement. However, if we arrange the statements of Exercise 7 on page 48 according to this plan we have:

Specific statement: Elizabeth is riding on the school bus.	General statement—Hypothesis: When Elizabeth goes home from school,
Conclusion:	Conclusion: she rides on the school bus.

Examination of these statements shows that the specific statement does not express the same thought as the hypothesis of the general statement; in fact, it more nearly parallels the conclusion of the general statement. For this reason no valid conclusion can be drawn.

If you use the scheme just described, make sure that you do not leave the impression that we use this arrangement when writing cycles of reasoning. This is a good time to point out that we usually write the general statement in the same line with the conclusion of the cycle since it is really the proof that the conclusion follows from the specific statement. Thus for Example 1 on page 46 we write:

Specific statement: Martha is a student who expects to graduate from high school.	General statement: If students intend to graduate from our high school, they must take three years of English.
Conclusion: She must take three years of English.	

This arrangement of a cycle of reasoning is discussed in §37.

Sec. 33 (p. 51). Proof in Geometry—In order to emphasize that once we have accepted a general statement and a specific statement, we must accept a conclusion regardless of our wishes, have the class set up a few nonsensical exercises with either false general statements or false specific statements, then determine the conclusion that must be drawn. Use such examples as

General statement: If a boy has big ears, he is bright.
Specific statement: John is a boy with big ears.

Conclusion:

Follow these nonsensical exercises by a serious discussion of the necessity for starting our system of geometric thought with statements that we are willing to accept as true if we wish our conclusions to be true. Make sure that the class sees that since we have no general statements that we can guarantee through proof, we must do the next best thing—rely on undefined terms, definitions, and any general statements we are willing to accept.

Sec. 34 (p. 52). Assumptions—As a matter of general information this section and §§35, 36 and 39 distinguish the assumptions that are axioms from those that are postulates. Throughout the remainder of the book, however, no distinction is made. We feel that the use of the word "assumption" instead of the words "axioms" and "postulates" not only simplifies the terminology of geometry for students, but also helps to keep before them the thought that geometry, instead of being a body of absolute truth handed down to us by authorities of the past, is really a living body of thought that changes as our assumptions change. This is in keeping with the most modern concepts of what geometry learning should be.

Sec. 35 (p. 52). Assumptions (Axioms)—Do not have students memorize these statements. Instead, work for complete understanding of the thought involved in each, and insist upon a complete and accurate statement of the thought. When thinking is right, the words fall into place naturally. Students should not be expected to remember which assumption is the first, which the second, and so on. Proofs in the textbook refer to the assumptions by number only to save space. Students should state the assumptions instead of giving their reference numbers.

Impress students with the thought that there is really nothing new or strange in these statements—that they are only statements which the students themselves have been accepting as true throughout their lives. Since they are statements accepted as true by practically everyone, they make good general statements to use in a proof—no one is likely to dispute a cycle of reasoning supported by one of them.

Before making an assignment from the list of exercises beginning on page 54, it is wise to have the class do two or three of the exercises to show just what is to be done in the assignment.

Sec. 36 (p. 55). Assumptions (Postulates)—The statements made for §35 apply equally well here. In discussing these assumptions, use plenty of chalkboard illustrations to add meaning. For example, in discussing Assumption 11, locate two points on the chalkboard and ask a student to draw a straight line through both. Ask him to draw another straight line through them. This forces the statement that through two points only one straight line can be drawn. Call attention to the fact that the same conclusion was reached in Exercise 3 of page 8 even before the class had studied the importance of assumptions.

With a very strong class you may want to point out that some of these assumptions can be proved true—for example, Assumptions 28-31. However, even though these statements can be proved, they are given as assumptions to simplify work for the class.

Sec. 37 (p. 59). Deductive Reasoning Arranged in Column Form—This section is designed to help students learn the form used for writing proofs. Emphasize that good form helps to make meanings clear.

Sec. 38 (p. 61). One-Cycle Proofs—In Example 1 the statement, "Prove that $x = 6$," should not be interpreted as saying, "Prove that 6 is a root of the equation $9x = 54$." The proof given shows only that if the equation $9x = 54$ has a root, then 6 is that root. To complete the proof that 6 is a root of the equation we need also to prove the converse statement: If $x = 6$, then 6 satisfies the equation. Example 1 is stated as it is in order to make the proof as simple as possible for beginners in geometry.

Make your assignment of the exercises on page 62 short enough to allow students to write the required proofs completely and accurately in the allotted time. At this stage in geometry learning quality is far more important than quantity. Follow the preparation of the assignment with careful checking so that each student knows the strengths and weaknesses of his proofs.

Chalkboard work is helpful at this point. By sending several students to the board to write proofs for exercises of this type you can oversee the work of several students at one time. Board work also enables each student to compare the quality of his proof with the quality of proofs written by other students.

Sec. 39 (p. 63). Assumptions (Axioms) Used in Solving Equations—Read again the first comment for §38. Do not let the treatment of the exercises on pages 64 and 65 become mere equation solving. Students should write complete demonstrations, including reasons, and should determine the number of cycles of deduction included in each proof.

Sec. 40 (p. 65). Definitions and Assumptions Used in Proofs—Insist upon complete and accurate statements. When in doubt about wording, students should turn to the page in the book on which the exact statement may be found. Show the class how to use the index for finding possible statements. For example, a student in doubt about Reason 4 should turn to the index (page 577) and look under "Angle" for "right." Since the page reference is "20" he should turn to page 20 and determine whether the reference is the one needed.

Secs. 42-48 (p. 68). Parts of a Formal Proof, . . . , How to Make a Formal Proof—These sections can be handled effectively by a chalk-talk procedure. First, write the theorem of §48 on the chalkboard and explain, as in §43, that a theorem is always stated as a complete sentence. Next, as in §44 and II of §48, explain how we draw a figure to represent the theorem. Draw the figure as you talk. As a third step, following the plan of §45 and III of §48, explain and write the "Given." If you follow this procedure until you have completed the proof, you have shown the class the model proof on page 49, and will have given the students suggestions on how to proceed in writing other proofs.

Sec. 49 (p. 70). Theorem 1—Point out the "Planning the Proof" in the completed proof on page 70 and explain that you did not write that in your chalkboard proof since it is really not a part of the demonstration. Explain that it represents the thinking that we must do before we can start writing a proof. Explain that the textbook includes "Planning the Proof" sections for most theorems, not because they are a part of the demonstration, but because they help provide students with a kind of model of how thinking can be organized to produce the particular proof being demonstrated.

Chapter 4. Triangles

Objectives:

To teach the concept of congruency.

To teach the simple relationships summarized on page 123 of the textbook.

To teach the demonstration of simple geometric proofs.

To develop self-reliance in proving original exercises.

To further demonstrate how the geometry the students are studying is taking shape as a logical system of thought.

Time required: 16-20 days.

Planning the content of various courses:

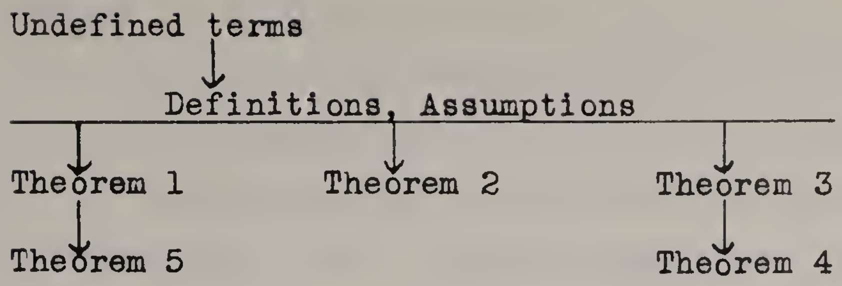
Basic course—§§54-85, 88-94. Because of time limitations students cannot be expected to study all the exercises of this chapter, but should do most of those on pages 76-101. Throughout the remainder of the chapter representative exercises should be chosen from each list. Theorem 2 should be proved and discussed in class. Students should prove Theorem 3 and be held responsible for being able to prove it on tests. Corollaries may be proved informally rather than formally, if desired. Theorems 4 and 5 may be proved and discussed in class or may be accepted without proof, but students should understand the ideas involved, and be able to use them.

Maximum course for students who plan to specialize in mathematics, science, engineering, and similar fields—§§54-94. In this course students should substitute B and C exercises for some of the A exercises. Some A exercises should be studied since many of them are designed to fix basic ideas; however, students should move through them quickly and use remaining time to concentrate on more challenging work. Theorems and corollaries should be treated as in the basic course except that students can be expected to do more with formal proof.

Minimum course—§§54-62, 65-68, study §§69-70 just before §84, §§73-85, 88-94. Accept Theorem 4 on page 110 and Theorem 5 on page 121 as true without proof. The other theorems may be treated as in the basic course. Students should do most of the A exercises on pages 76-101 and some A exercises from each list from page 102 to the end of the chapter. Encourage students to attempt some of the simpler B exercises.

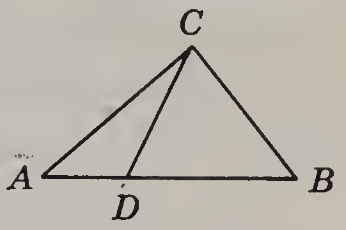
Specific suggestions for teaching the various topics:

By now the students should have accumulated a set of basic definitions to use in building the system of geometric thought, have agreed to accept certain assumptions, and have learned the fundamentals of deductive reasoning. If so, they are now ready to begin the actual building of the system. The outline at the top of the next page shows the structure of the system as it will appear when students have completed Chapter 4. We are presenting the completed outline here so that from time to time you can show students the sequence of the theorems of the chapter. To keep the outline simple we have not included corollaries and exercises. Neither have we attempted to show the contributions of undefined terms, definitions, and assumptions beyond Theorem 3. However, you will have no difficulty in supplying the missing relationships if you wish to make the diagram more complete.



Sec. 54 (p. 77). Polygon—Instead of making this and succeeding sections of pages 77-80 part of an assignment, you may want to save time by summarizing the thoughts involved. Make sure that the students understand that by "polygon" we mean the broken line, and not the space enclosed by the line. Do not take for granted that students will understand the meaning of "consecutive angles" and "adjacent sides."

Sec. 55 (p. 77). Triangles—Emphasize that a triangle has only six parts—three sides and three angles. Students have a tendency to overlook this thought, then later to begin calling any angle or line enclosed by the triangle a part of the triangle. For example, in the △ ABC shown at the right, they call line CD and $\angle$s ADC and BDC parts of △ ABC.



Secs. 56-57 (p. 77). Triangles Classified as to Sides, Angles —Students should be familiar enough with the various classifications to use the proper descriptive word without hesitation.

Sec. 58 (p. 78). Parts of an Isosceles Triangle—Students should notice the word "usually" in the first sentence of this section. If an isosceles triangle rests upon one of its equal sides as a base, the expressions "vertex angle" and "base" as used here lose their meaning.

Sec. 59 (p. 80). Included Sides and Included Angles—In a class there are sometimes two or three students who have difficulty with the meaning of "included angle" and "included side." Explain that included means "enclosed by." When two lines form an angle, they enclose it. The angle is thus included between the lines. Similarly, a side is enclosed by the two angles that have it as their common side. The side is included between the angles. Draw several polygons on the board, letter their vertices, and ask such questions as, "What angle is included between sides AB and BC?" "What side is included between angles A and B?"

Sec. 60 (p. 80). Parts of Right Triangle—Students should notice that "leg" and "hypotenuse" are special names for the sides of right triangles only.

Sec. 61 (p. 80). Common Sides and Common Angles—Ask students who have difficulty with the meaning of "common side" and "common angle" what we mean when we say that Bill and John have a bicycle in common. Use chalkboard illustrations to show examples of included sides and included angles. The exercises of pages 80 and 81 help to develop these meanings.

Sec. 62 (p. 81). Congruent Figures—You may want to explain that the symbol "≅" is made up of two symbols, the symbol "=" meaning "equal" and the symbol "∼" meaning "having the same shape." Thus the symbol denotes that congruent figures have the same size and the same shape. Students may be interested in knowing that the symbol "∼" is an s laid on its side. The s stands for "similar"; the descriptive adjective used for two triangles having the same shape. It is easy to illustrate congruent figures by holding two sheets of paper together as you cut a triangle from them. Point out that the resulting triangles coincide, that is, fit together. Comment that the angles which fit together are corresponding angles and the sides which fit together are corresponding sides. Teach your class the proper pronunciation for the word "congruent."

Sec. 63 (p. 82). Proof by Superposition—It may be that you will not want to have all your students study this section and the next. However, students who plan to continue the study of mathematics should become familiar with the concept of superposition.

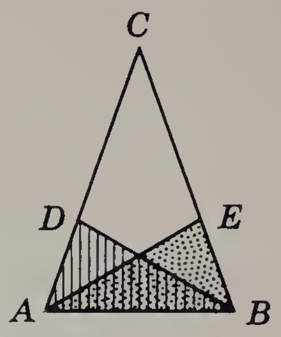
Sec. 64 (p. 84). An Investigation—For the benefit of the students in your class who will continue the study of mathematics you will probably want to discuss this investigation. By drawing one of the triangles in colored chalk, it is easy to show the coincidence of parts of the triangles. For example, if triangle ABC is drawn with white and triangle DEF with colored chalk, you can ask, "If I should pick up triangle ABC and place it on triangle DEF, can you tell me one pair of parts that you can guarantee will fit together?" Obviously, the student may answer, "∠A on ∠D," "∠B on ∠E," or "AB on DE." Regardless of which of these answers the student gives, you can then overlay the colored line segment or angle with the white line segment or angle. For example, if his answer is "AB on DE" you can draw a white segment over the colored segment DE and label the new segment AB after determining that the student is sure that A falls on D and B on E. Proceeding in this way you can eventually show that the two triangles have coincided throughout and have therefore met the requirements for congruency.

Follow through by asking whether the triangles could have been shown congruent had it been given that ∠ A = ∠ D, AC = DF, and ∠C = ∠ F? if ∠C = ∠F, BC = EF, and ∠B = ∠E? When the class is convinced that the procedure would give the same results for these relationships, ask whether a change in the size or shape of the triangles would affect the result, provided that one set of these relationships remains in effect. If the students can be shown that the congruency remains invariant under all these changes, they are ready to accept Assumption 32.

Sec. 65 (p. 84). Assumption 32—Regardless of whether you proceed as indicated in §64 of the textbook, you should make clear that irrespective of the shape and size of the triangles, so long as two sides and the included angle of one triangle are equal to two sides and the included angle of the other, the triangles are congruent. In these discussions make sure that the students are following the thoughts expressed. This assumption is so very important that you should not proceed without complete student understanding of what is involved. Point out that to avoid writing the long statement of the assumption we often use the abbreviation S.A.S. Students may be allowed to use the abbreviation so long as you are convinced that they know its meaning.

Sec. 66 (p. 84). How to Use Assumption 32—After accepting Assumption 32, students sometimes want to broaden it to include other relationships--for example, that two triangles are congruent if three sides of one are equal to three sides of the other. Remind such students that they have not yet studied these other cases and are therefore in no position to begin making assumptions about them, that they will eventually study the other cases, but at present must confine their attention to the one assumption they have agreed is true. Reinforce your statement with a chalk-talk procedure in which you hastily draw on the chalkboard several pairs of triangles with tic marks to indicate which parts are to be considered equal. Use the S.A.S. relationship for some pairs and the still unconsidered S.S.S., A.S.A. and A.A.A. relationships for others. As you rapidly draw these pairs of triangles, ask students to indicate whether the triangles of a pair can be said to be congruent. Exercises 1-10 of pages 85-86 in the textbook give similar practice. You may need to give extra time to this topic as it is useless to proceed to new work until students have grasped the concept that triangles are congruent when two sides and the included angle of one are equal respectively to two sides and the included angle of the other, and also the concept that no other case of congruency (except the definition for congruency) has been studied.

Sec. 67 (p. 87). Overlapping Triangles—One good way to designate overlapping triangles is indicated at the right. By shading one triangle black and covering the other with dots neither triangle overshadows the other. Moreover the corresponding sides and angles are clearly visible. The procedure does have one weakness in that it leaves the impression that a triangle is a portion of a plane rather than a broken line. If you are careful, however, this handicap is not great. Triangles cut from felt or paper and laid to overlap also help students to visualize the situation, but such triangles have the same shortcoming as the pencil shading described above.



Sec. 68 (p. 88). Using Assumption 32 in Proofs—You will probably want to introduce this list of exercises through a chalk-talk class-discussion procedure in which step-by-step you work out on the chalkboard a complete proof similar to those on page 88 of the textbook. As you write each step you will want to explain and question. Your students will tend to imitate your carefulness. It you write complete well-organized proofs, so will they.

If there is time left in the class period after the demonstration, you may want to send a group of students to the chalkboard to write proofs for Exercise 1 on page 89. Other students can work at their seats. Having some of the work done at the board has several advantages--you can oversee more students in a given amount of time, students try harder because they do not want other students to see their shortcomings, and the completed proofs provide examples which help you to direct class attention to particular points.

When you are sure the class knows how to proceed, assign two or three of the exercises on page 89 as homework preparation for the class next day. Keep your assignment short--so short that even the poorest student can succeed. Your greatest ally at this point is a student's confidence in himself. Make sure, during these first lessons on proof writing, that students can succeed.

In class next day every paper should be checked. One effective method of checking is to write a model proof on the chalkboard as students dictate, and discuss the statements and reasons to be written. Do not let it worry you that the checking will take most of the class period. Your concern should not be with the number of exercises covered, but with student understanding. Checking directs attention to points of difficulty. It also inspires careful preparation.

It is wise, at this point, not only to provide students with a check on the correctness of their proofs, but to give them a definite measure of their success. This may be done by allowing one point for a figure correctly drawn, one for the "Given" correctly copied, one for the "To prove" correctly copied, one for each correct statement, and one for each correct reason in the proof. In general, the number of correct statements and reasons is not a good measure of the quality of a proof, but it serves the purpose at this time. Many geometry students feel lost with the vagueness of geometry (after having studied algebra with its self-checking characteristics) and are relieved to find a way to measure their efforts. More students fail in geometry because of a feeling of insecurity than through lack of intelligence or lack of study.

Make the next assignment two or three more of the exercises on page 89. Check these carefully when they are brought back to class. If possible, supplement this "homework" with classwork at the chalkboard. By sending several students to the board to write proofs of a dictated exercise, you can provide individual help for several students within a short period of time, and can identify students who are obtaining enough outside help to make a creditable showing on "homework" even though they cannot succeed alone.

Secs. 69-70 (p. 90). Corollaries, Proof of a Corollary—If you plan to omit §71, you should postpone study of §§69 and 70 until students are ready to prove the corollary in §84. Regardless of when §§69-70 are studied, make clear that a corollary obtains its proof from the theorem, assumption, or other corollary which precedes it.

If you teach the corollary in §71, emphasize that its hypothesis is a special case of the hypothesis of Assumption 32.

Secs. 72-74 (p. 91). An Investigation, Assumption 33, How to Use Assumption 33—You will probably want to handle these sections by methods similar to those described for §§64-66.

Sec. 75 (p. 93). Using Assumption 33 in Proofs—Discuss at least one of the examples on pages 93-94 thoroughly before you make an assignment from the exercises on pages 94 and 95. Keep your assignment short and in return insist upon complete and carefully prepared proofs. Even average students should be able to manage Exercises 3, 4, and 5 on page 95. An assignment for an average class might consist of Exercises 1, 2, and 4. After that assignment has been checked and discussed, you may want to prove

Exercise 6 as a class project with all students free to participate. (This exercise necessitates some new thinking.) Your next assignment might require written proofs for Exercises 3 and 5 of page 95 and Exercise 1 of page 96. (See note for § 76 below.)

Sec. 76 (p. 95). Using Assumptions 32 and 33—Before assigning the exercises on pages 96-97, either discuss § 76 or make it part of the first assignment. Without this preparation students may be puzzled to find some exercises that depend upon Assumption 32 and some that depend upon Assumption 33.

Continue to make assignments short and continue to check student work. By now you will probably need to begin to vary your methods of checking. On some days you might ask all the students in one row to write their proofs for the first exercise in the assignment on the chalkboard, after which the class can point out strong and weak points in the various proofs. A second row might write another exercise, or one student might be asked to do so. On other days you may want to collect the papers and check them yourself.

One effective way to handle exercises (particularly the more difficult exercises) is to say to the class, "Study Exercise 7 (or any other exercise you select), then as soon as you think you can prove the exercise go to the board and write your proof." If you have a chalkboard at the back of the room use it in order to prevent copying. If you do not have enough chalkboard space (you probably will not have enough since students like this procedure and respond to it well), have the remaining students write their proofs on paper to be checked by you later.

Even average students should be able to prove the B exercises on page 97, especially if you prepare them for the assignment with a few suggestions. Exercise 11 is also easy although it is a C exercise. It therefore becomes good for morale building—average and weak students are delighted to find that they can manage some C exercises.

If your class is of more or less uniform ability, the handling of the more difficult exercises is not a problem—either the class studies them or it does not. However, if you have within the class strong students, average students, and weak students, finding ways to challenge the strong, without discouraging the weak, becomes a problem. One solution is to give differentiated assignments. Another is to give extra credit for extra work on the more difficult exercises. Dividing the class into small groups, each group to collaborate on the solving of one exercise, is helpful occasionally.

Sec. 77 (p. 98). Corresponding Parts of Congruent Polygons—Students were introduced to the idea of corresponding parts in §62, but it is best to discuss the topic again and to answer some of the questions asked in the oral exercises on page 98.

Sec. 78 (p. 98). Assumption 34—If students have been reasonably successful in proving the exercises on preceding pages of this chapter, they will have little difficulty with pages 99-101. Do not neglect these exercises, however, for they help to build self-confidence and to increase the student's power to think analytically. Use the B and C exercises to challenge your better students.

Sec. 79 (p. 102). Theorem 2—You may want to discuss §82 at this time. Emphasize the reason for Statement 1 of the proof. Call attention to the fact that Statement 3 is a consequence of Statement 2 and that Statement 5 is a consequence of Statement 4; in other words, we cannot give Statements 3 and 5 unless we have first given Statements 2 and 4. In writing proofs students are inclined to omit such statements as 2 and 4.

Sec. 80 (p. 103). Meaning of the Word "Any"—In explaining the meaning of the word "any" it may help to explain that since all of us cannot meet to decide upon needed state and national laws, we select senators and representatives for state legislatures and Congress. Similarly, since we cannot prove that every point on the perpendicular bisector is equidistant from the ends of the segment, we choose a point which represents all of the others.

Secs. 81-83 (p. 103). Directions for Proving Theorems, Auxiliary Lines, Theorem 3—After students have studied §§81-83, you may find it worthwhile to draw another figure for Theorem 3—one in which the vertex angle is either much larger or much smaller than $\angle C$ of the figure in the textbook, with its vertices named by other letters—then ask the students to prove that the angles opposite the equal sides are equal.

Sec. 84 (p. 106). Corollary—In proving a corollary students have a tendency to want to use the same auxiliary lines that were used in proving the theorem from which the corollary is derived. If your class suggests using the auxiliary line used in Theorem 3, do not object. Let the students find their way through the proof, then when the proof is complete ask if anyone recognizes a way to shorten it. It is almost certain that someone in the class will recognize the possibility of saying (for an equilateral $\triangle ABC$), "Since AB = BC, by Theorem 3 we know ∠ A = ∠ C, and since AB = AC, by Theorem 3 we know that ∠ B = ∠ C; therefore by Asmt. 7 we know that ∠ A = ∠ B = ∠ C."

Most of the exercises on pages 107-109 provide practice in using Theorem 3 and its corollary to prove new geometric relationships. Your class should study as many of these exercises as time permits. While your assignments can now become a little longer, you should still make sure that they are not so long that students do not have time to write each assigned proof completely and carefully.

Sec. 85 (p. 110). Theorem 4—Since the two triangles described in Theorem 4 cannot be placed together as shown unless they are drawn in a particular position with respect to each other, or else one triangle is taken out of the plane and turned over before being placed back in the plane, many teachers prefer to have students accept the theorem as true without proof. Students have good imaginations, however, and it does them no harm to realize that we must resort to space geometry. This is particularly true today when many schools are teaching plane and solid geometry as an integrated course.

This theorem gives you an opportunity to show your class how the system of geometric thought is beginning to take shape. Use the outline given on page 41 of this manual.

While you probably cannot spare enough time for your class to study all the exercises on pages 111-114, do as many as possible. To a large extent, exercises rather than theorems develcp the power of students to think analytically.

Secs. 86-87 (p. 115). Syllogisms, Euler's Circles—While these sections are marked "Optional" use them if possible. They increase understanding of the nature of deductive reasoning.

Secs. 88-92 (p. 118). Line Segments and Angles Associated with Triangles, Medians of a Triangle, Altitudes of a Triangle, Angle Bisectors of a Triangle, Exterior Angles of a Triangle—Make sure that students know the exact meaning of each of the italicized words in these paragraphs and insist that when the definitions are used, they be stated completely and accurately.

The exercises on pages 119 and 120 emphasize relationships between these line segments and angles. Assign as many of them as time and ability permit.

Sec. 93 (p. 121). Theorem 5—Many teachers allow students to accept the statement of this theorem as true without proof. If you wish to have students study pages 504-507 of the textbook in connection with this theorem it is entirely possible to do so. The book is so arranged that pages 504-507 do not depend upon topics which your class has not yet studied.

Chapter 5. Parallel and Perpendicular Lines

Objectives:

To teach the concept "parallel" and to lead students to prove the relationships summarized on pages 179-180.

To introduce indirect proof and analytic proof.

To teach the meaning of "converse," "inverse," and "contrapositive," and to point out their usefulness.

Time required: 16-20 days.

Planning the content of various courses:

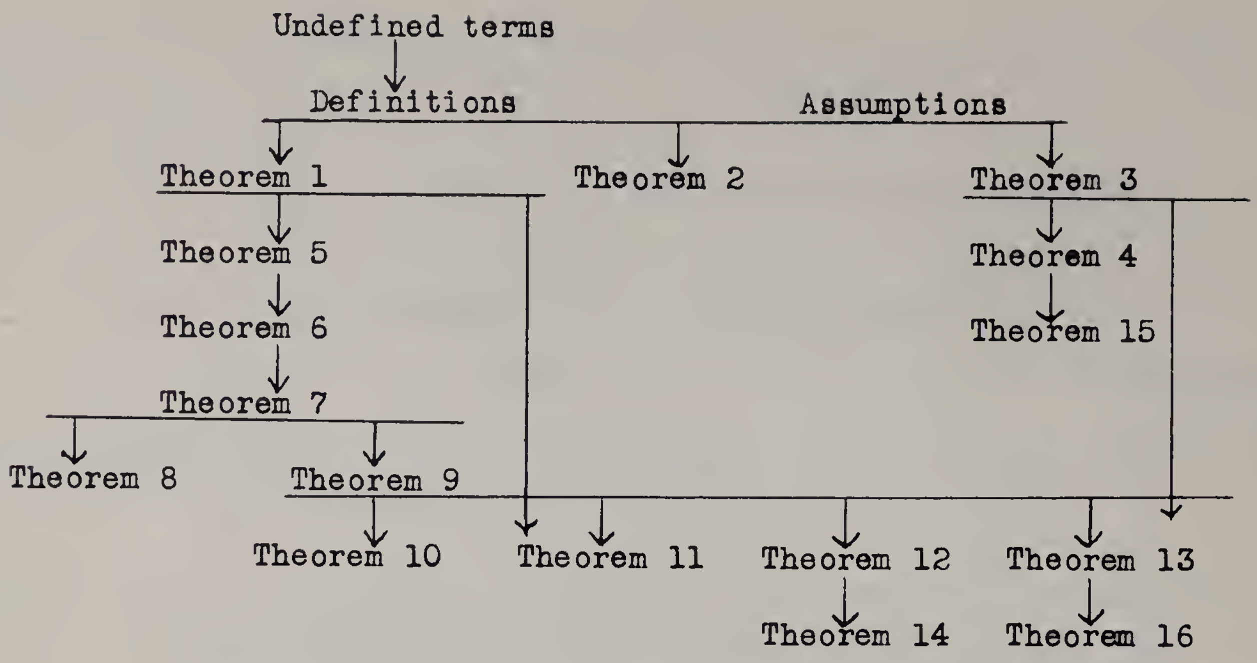
Basic course: §§99-117, 120-143; 144-155 optional. Except for Theorems 6 and 7, students should be held responsible for ability to prove on tests all theorems and corollaries marked with two stars. All theorems and corollaries marked with one star should be proved in class. Theorems 6 and 7 and unmarked theorems and corollaries may be proved informally. No class will have time to do all the exercises in the book, but average students should study as many A exercises and easier B exercises as time permits.

Maximum course for students who plan to specialize in mathematics, science, engineering, and similar fields: §§99-143. §§144-155 as desired. Theorems should be treated as suggested for the basic course. In the maximum course emphasis should be upon ability to solve B and C exercises as well as A exercises. No student can be expected to do all the exercises in the chapter.

Minimum course: Use the outline suggested for the basic course with the following changes: Develop the content of §102 more slowly (over a longer period of time); accept as true without proof (or with the most informal of proofs) Theorem 6, the corollaries in §§109 and 110, and Theorem 7; spend less time on §137; omit §142. Students should be able to do most of the A exercises.

Specific suggestions for teaching the various topics:

From time to time show students the structure of the geometric system they are building. The outline at the top of the next page shows the sequence of theorems as it will appear when students have completed Chapter 5. When the matter is discussed before the chapter is completed, use only the part of the outline which shows work already completed. We are presenting the complete outline at this time in order to help you in your planning.



To keep the outline simple we have not included corollaries and exercises, and with two exceptions, we have not shown the contributions of undefined terms, definitions, assumptions, and previously proved theorems. We have shown that both Theorem 1 and Theorem 9 contributed to Theorem 11 and both Theorem 3 and Theorem 9 contributed to Theorem 13. As the system is outlined, a theorem derived from a corollary is shown as being derived from the theorem which produced the corollary.

Secs. 99-100 (p. 129). Parallel Lines, Assumption 35—You can show your class the significance of the words "if they lie in the same plane" by holding two rulers so that one runs east and west and the other north and south but above the first ruler. Point out that though the lines represented by these rulers "do not meet even if extended" they are not parallel since they do not lie in the same plane. These lines, called *skew lines*, are studied in solid geometry.

Sec. 101 (p. 129). Angles Formed by a Transversal—Students should be familiar enough with the italicized expressions in this section to use the expressions correctly and without hesitation. Teach your class the proper pronunciation of the word "alternate." Students have a tendency to pronounce the word as if it were a verb.

Secs. 102-103 (p. 130). Indirect Method of Proof, Assumption 36—Since many students find indirect proof difficult at first, be particularly careful in teaching this section. Point out to your class that in our everyday thinking all of us make much use of indirect reasoning. You may want to supplement the book examples with such examples as the following:

Mary said to her mother, "I shall either carry my umbrella or wear my raincoat." Later when Mary's mother saw the umbrella where Mary had left it, she decided that Mary had worn her raincoat.

Show the class how the steps "To prove a theorem by the exclusion method" in the box on page 131 may be applied to this example. To do so point out that:

1. In the first place Mary's mother reasoned: Either Mary wore her raincoat or she carried her umbrella. Mary mentioned no other possibilities.

2. Next Mary's mother reasoned: If Mary carried her umbrella, it would not still be here at home. However, the umbrella is here, and this fact refutes the possibility that Mary carried the umbrella with her when she left.

3. Finally, Mary's mother reasoned: Since Mary gave me only two possibilities and one of these is known to be false, the other must be true, that is, I must conclude that Mary wore her raincoat.

Sec. 104 (p. 132). Theorem 6—First lead your class to analyze the thought expressed in the theorem and to discuss ways in which the conclusion of the theorem may be proved true. Lead students to see that since none of the usual methods of proof seem helpful here, indirect proof should be tried.

Have the students turn to the procedure outlined in the box on page 131 and observe that Step 1 of the procedure suggests Statement 1 of the proof, namely "ℓ and ℓ' are either $\parallel$ or intersecting lines." Next have them observe that Step 2 of the procedure leads to Statements 2, 3, and 4 of the proof. Finally, point out that Step 3 of the procedure leads to Statement 5 of the proof.

Follow discussion of the proof with chalkboard practice in which you draw on the board pairs of lines in many positions, each pair being so related that when the two lines are crossed by a transversal the alternate interior angles are equal. For each pair point out the equal angles and ask the class what consequence follows from the fact that the alternate interior angles are equal.

Secs. 105-107 (p. 132). Corollaries I, II, III—Students tend to believe that since they used an indirect proof for Theorem 6 they should also use indirect proofs for the corollaries. You can easily start class thinking in the right direction by the questions you ask. For example, before studying the corollaries ask the class to list the ways it now has for proving lines parallel. Students should point out that there are two—§§99 and 104. After considering this fact most students will recognize that they can easily prove the corollaries by proving the alternate interior angles equal—a task that is easy enough in each case.

Choose four or five exercises from page 134 for class study.

Sec. 108 (p. 134). Assumption 37—This section provides a good example of how changing our assumptions changes our conclusions. Euclid made the assumption given in §108; Lobachevsky, Bolyai, Reimann, and other mathematicians made other assumptions about the number of lines that can be drawn through a given point and parallel to a given line. Each man then arrived at conclusions consistent with his assumption, though not necessarily consistent with the conclusions reached by the others. Point out that it is wrong to say that Euclid's geometry is true and the others false, or that one of the others is true and Euclid's false. Each is correct for its particular assumptions. Euclid's geometry seems to describe the world as we know it, but perhaps when we know more about all space, we shall find that one of the other geometries provides a better description.

Secs. 109-110 (p. 135). Corollaries I, II—Only strong students will be able to prove these corollaries without help from you.

Sec. 111 (p. 136). Theorem 7—In proving this theorem follow a plan similar to that suggested for Theorem 6, §104. Be sure to have students start by analyzing the thought of the theorem and then to think through how best to arrive at a proof.

Secs. 112-115 (p. 136). Corollaries I, II, III, IV—In proving these corollaries follow the suggestions for the corollaries of §§105-107.

Students should study at least half of the exercises on pages 137 and 138. You may want to point out that Exercise 5 is really a corollary to Theorem 6. Exercises 6, 7, 8, 10-15 prepare students for later theorems. Both C exercises are valuable for strong students.

Sec. 116 (p. 138). Right Side and Left Side of an Angle—Use of the expressions "right side" and "left side" instead of "initial side" and "terminal side" simplifies thinking for the students and allows them to give their undivided attention to the thoughts expressed in the theorems. Otherwise, much student attention is diverted to recalling which side is the initial side and which the terminal. The expressions "right side" and "left side" are adequate for a first course in plane geometry.

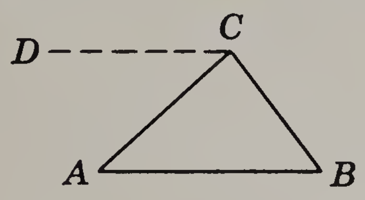
Sec. 117 (p. 139). Theorem 8—Students tend to fear this theorem at first, probably because it looks complicated. Mention that the proof is easy. Lead the class to analyze what is given and what to be proved, then ask students to list ways in which angles can be proved equal--particularly ways which involve parallel lines. From this point the students will be able to discover several possible proofs. Encourage original thinking by being free with your praise for students who originate proofs other than the one in the textbook.

In the exercises following the theorem proceed according to the suggestions made for the theorem.

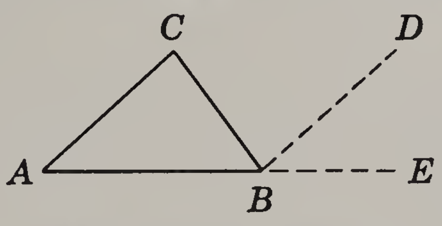
Sec. 118 (p. 139). Parallel Lines in Television—This section is included as enrichment material, and is not essential to understanding later sections of the textbook.

Sec. 119 (p. 141). Indirect Reasoning in Life Situations—This topic can be used as a "filler" for a class period when there is time left after the regular class work is completed.

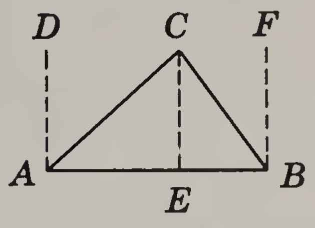
Sec. 120 (p. 142). Theorem 9—To stimulate original thinking ask students to see how many proofs of this theorem they can discover, or give an easier assignment by suggesting possible auxiliary lines and asking students to give the appropriate proof for each set of lines. The following are among the possibilities.



Construct DC||AB



Extend AB to E and
construct BD||AC



Construct DA, CE, and
FB ⊥AB

Secs. 121-125 (p. 142). Corollaries I, II, III, IV, V—Emphasize that since a corollary obtains its proof from the theorem which precedes it, in each of these corollaries we may start with the statement, $\angle A + \angle B + \angle C = 1$ st.$\angle$ (provided, of course, that the triangle is named with the letters A, B, and C).

In choosing exercises for class study from those on pages 143-144, be sure to select some that require proof as well as those that are merely numerical exercises or questions to be answered.

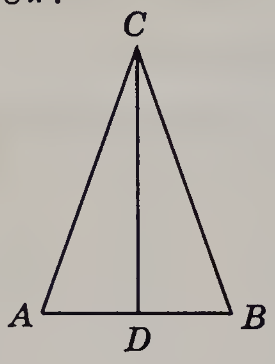
Sec. 126 (p. 145). Theorem 10—It may be necessary for you to call attention to § 15 once more. Otherwise, students find this proof easy.

Sec. 127 (p. 146). Theorem 11—Students have a tendency to think they should prove triangles congruent. Forestall this by drawing a figure in which $\triangle$ BOH is much smaller than $\triangle$ OEP, or vice versa. As an original exercise ask the class to prove the theorem by use of § 121.

In proving the exercise at the bottom of page 146, lead students to see that if PE of the figure at the top of the page is extended downward to L, the resulting $\angle FEL$ is supplementary to $\angle FED$. Since $\angle FED$ has already been proved equal to $\angle B$, the proof for the exercise follows easily.

Secs. 128-129 (p. 147). Theorem 12, Corollary—Students find these proofs easy. Choose representative exercises from pages 147-148.

Sec. 130 (p. 149). Converse Theorems—To help your students visualize what is meant by interchanging any number of conclusions with an equal number of hypotheses write the theorem, "The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to the base," in "Given...To prove..." form as shown below.



Given $\triangle ABC$ in which $AC = BC$,
CD bisects $\angle C$.

To prove that CD bisects AB,
$CD \perp AB$.

It is now easy to show the class that both statements of the "Given" (we do not count $\triangle ABC$) may be interchanged with both statements of the "To prove" to produce one converse, and each statement of the "Given" may be interchanged with each statement of the "To prove" to produce four other converses. Write each converse as it is discussed.

Secs. 131-134 (p. 150). Inverse Theorems, Contrapositive of a Theorem, Equivalent Statements, Assumptions 38, 39—Teach these sections carefully for considerable use is made of them in later sections of the textbook. Make sure that students understand the relationships explained at the top of page 153.

Sec. 135 (p. 154). Theorem 13—Students often overlook important points in this proof. For example, many fail to see why it is important that ABD be a straight line. If one of your students makes this error, draw ABD as a broken line and ask the student to proceed with the proof. He soon sees that unless ABD is a straight line segment, the figure ABDC is a quadrilateral, and as a result, we cannot find a way to prove ∠A = ∠D. Do not let students substitute memorization of the proof for understanding of it.

Sec. 136 (p. 155). Theorem 14—This theorem is particularly important in applied geometry. Do not neglect it. Careless students sometimes state that in any right triangle the side opposite the smaller angle is one half the hypotenuse.

Sec. 137 (p. 156). Analytic and Synthetic Methods of Proof—In assigning exercises from pages 157-159, do not use so many that the students become weary of the procedure. One or two lessons will suffice. In future difficult exercises urge students to analyze the problem before they attempt to write the proof.

Secs. 138-139 (p. 159). Proof by the Coincidence Method, Theorem 15—Be prepared to explain that the proof in Theorem 15 is indirect in that we work not with the given perpendicular bisector CD, but with a line PC which we add to the figure. By proving that the auxiliary line PC is the perpendicular bisector, we know that PC must be the same line as the given line CD. If students are troubled about having two forms for indirect proof show them that this proof could be demonstrated as a proof by exclusion; we use the form which seems most natural for the particular situation.

Sec. 140 (p. 160). Corollary—Students have a tendency to misquote Theorem 15 through confusion with the corollary in §140. They say for Theorem 15: A line passing through a point equidistant from the ends of a segment is the perpendicular bisector of the segment. Point out to such students that they are stating a converse of Theorem 15. This converse happens to be a converse which is not true. Demonstrate with illustrations on the board that an infinite number of lines can be drawn through a point equidistant from the ends of a line segment. The only one of these which can be said to be the perpendicular bisector is the one mentioned in §140—the line which passes through not only that point but another point equidistant from the end points of the line segment.

Sec. 141 (p. 161). Theorem 16—This proof is indirect in that we do not work with the given angle bisector BF. Instead, we draw a line BP and work with it until we show that it is the same line as the line BF.

Sec. 142 (p. 165). Symmetry—This topic is included as a matter of general information. Omitting it will not obstruct further work in plane geometry.

Sec. 143 (p. 166). Location of Points—This topic prepares students for the study of loci in Chapter 11. Anything you can do now to begin developing the concept of locus will be of great help later.

Secs. 144-155 (p. 172). Space Geometry—These sections are included for teachers wishing to integrate plane and solid geometry. These sections, and those of later chapters, are placed at the ends of the chapters to prevent complicating matters for teachers who do not want to include them in the plane geometry course.

Chapter 6. Constructions

Objective:

To teach students to analyze geometric relationships through use of compasses and a straightedge.

Time required: 4-5 days.

Planning the content of various courses:

Basic course—§§163-175, §176 optional. Students should be held responsible for ability to (1) make the constructions for the problems marked with two stars, (2) prove that the constructions made are correct, and (3) discuss the special cases related to the problem. Assignments should include some of the A and B exercises of each list in the text.

Maximum course for students who plan to specialize in mathematics, science, engineering, and similar fields—Same as the basic course except that students should do the more difficult B exercises and some of the C exercises.

Minimum course—Same as the basic course except that assignments of exercises will need to be confined mostly to A groups.

Specific suggestions for teaching the various topics:

Sec. 163 (p. 185). Procedure in Making Constructions—An architect or a machine draftsman makes drawings that are to be used. His interest is in the drawings as drawings. In geometry, however, we are not interested in completed constructions as objects of art or usefulness; we are interested in the thinking process which produced the drawing. The emphasis in teaching this chapter should be upon careful analysis of problem situations and the discovery of ways to solve the problem using only compasses and a straightedge. Since the construction problems of the textbook are to be exercises in thinking, students should keep their work organized and clear. They can do this best by following a set plan such as the conventional one outlined in the box on page 185.

Emphasize the last paragraph of § 163. It answers questions about why we say our constructions are correct.

Sec. 164 (p. 185). Proposition, Theorem, Problem—Many books refer to the various construction problems as Construction 1, Construction 2, and so on. We have chosen to refer to them as Problem 1, Problem 2, and so on. We feel that this emphasizes in the minds of students the thought that they are solving problems, even though the solutions are carried out by use of compasses and a straightedge.

Sec. 165 (p. 186). Problem 1—Since this is the first construction problem (since the one given informally in §9), you will probably want to carry out the process on the board as the class refers to the book, offers suggestions, and asks questions. Start by drawing on the board three line segments a, b, and c which, according to the statement of the problem, are to be the three sides of the triangle to be constructed. (Make sure you draw three which will form a triangle. Special cases can be discussed later.) Ask students to state the "Given" and the "To construct." Write them on the board as the students state them. Your carefulness in such matters sets the pattern for later student work. Now ask for suggestions about how to proceed in making the drawing, and be ready to accept any correct procedure even though it may not be exactly like the procedure in the text. As each step is given write it on the board in a section headed "Construction." Ask for and write a reason to back up each step of the construction. Be sure to point out that we always end the description of the steps in the construction with a statement that the completed figure is the required figure (the step in italics following Step 4 in the demonstration in the text.) Show that we must next prove that this last statement in the description of the construction is true. Write the heading "Proof" and proceed to write the proof even though it is very short. Mention that proofs are not always as simple. Finally, lead the class to suggest various special cases that could be obtained—the case in which the given segments do not form a triangle, the case in which the triangle is isosceles, and so on.

Exercises 2 and 4 of the list following the problem mention measurements to force students to think through special cases. You may prefer to have students draw a line segment to use as a unit to replace the inch, thus avoiding reference to measuring devices. In this case the three sides of the triangle would be $1\frac{1}{2}$ times the segment drawn, 2 times the segment, and 3 times the segment.

Secs. 166-167 (p. 187). Problems 2, 3—Whether the students in your class study these problems as a group or as individuals, make sure that every step of the process is carefully and completely carried out.

Sec. 168 (p. 188). Construction Problems in Proofs—Do not overlook this section. It serves two purposes. It states that one construction may become the reason for supporting a succeeding construction and it clarifies the wording of problems so used.

Secs. 169-170 (p. 189). Problems 4, 5—Ask the class what would be true of all the triangles drawn if each member of the class used the given parts in Problem 4. Make sure students see why all the triangles are congruent. Repeat for Problem 5.

Secs. 171-173 (p. 190). Problems 6, 7, 8—One good procedure is to make and discuss the three basic constructions in class and to use the exercises following each problem as an assignment to be prepared by students. You may wish to excuse students from writing out complete proofs and descriptions of the steps in the construction provided they can give the missing parts upon demand. Writing complete demonstrations for all construction problems becomes tedious and kills student interest. You can judge correctness of a construction by studying the arcs and lines in the drawing (constructions lines and arcs should not be erased).

Sec. 174 (p. 193). Problem 9—It is good to ask students to see how many ways they can find for making this construction. Two acceptable ways are mentioned in the exercises at the bottom of the page, but more are possible.

Sec. 175 (p. 194). Directions for the Solution of Construction Problems—After studying page 194 carefully students should be able to do all the A exercises on page 195 and the better students should be able to do the B and C exercises. It is probably best not to ask for written descriptions of the steps in the construction of such exercises so long as you are sure students can, when necessary, produce the descriptions.

Sec. 176 (p. 196). Trisection of an Angle—This topic is included for enrichment purposes. Only the more able students will be interested in and able to follow the description, "Trisecting an Angle Using the Conchoid," but all will be able to understand the devices described in Exercises 1 and 3 of page 197.

Be prepared to explain why joining the points of trisection of the base of an isosceles triangle to the vertex does not trisect the angle, since this is an "invention" of a few students in every class. Even though you are prepared to explain why the construction is incorrect, it is usually best to leave to the student, himself, the task of showing that his construction is correct or incorrect. Be prepared to help, however.

Chapter 7. Polygons

Objectives:

To develop in students additional power in using deductive reasoning to discover geometrical relationships.

To teach the relationships summarized on pages 233-234.

Time required: 12-16 days.

Planning the content of various courses:

Basic course—§§182-212, 215. §§213-214 optional. Students should be held responsible for ability to (1) prove on tests all theorems and corollaries marked with two stars, and (2) make, prove, and discuss on tests the construction for Problem 10. Theorems and corollaries marked with one star should be demonstrated and discussed in class. Those which are unmarked may be done informally. No class can be expected to do all of the exercises in the chapter but should study as many of the A and easier B exercises as possible.

Maximum course for students who plan to specialize in mathematics, science, engineering, and similar fields—Same as the basic course except that students should do more of the B exercises and some of the C exercises.

Minimum course—§§182-212, 215. Theorems and problems should be treated as in the basic course. Exercises for this course will need to be chosen mainly from A groups.

Specific suggestions for teaching the various topics:

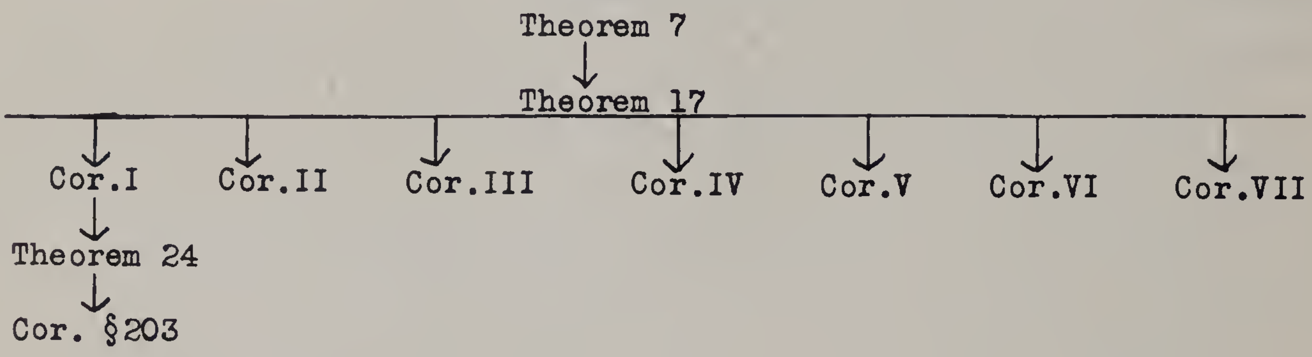
Sec. 182 (p. 201). Polygon—Make clear that a polygon is a broken line, not the space enclosed by the line.

Secs. 183-184 (p. 201). Polygons Classified as to Angles and Sides, Regular Polygon—Students should know the meanings of all the italicized words in these sections and should be able to give the names of the various polygons mentioned in the table at the bottom of page 201.

Sec. 185 (p. 202). Parallelogram—It is very important that students have in mind the definitions for the various special kinds of quadrilaterals because in proving a quadrilateral a parallelogram, rectangle, etc., it is necessary to base the proof upon the definition of that particular figure, or upon another proof which is based upon the definition.

Sec. 186 (p. 202). Base and Altitude of a Parallelogram—Parallelograms are so often drawn with the longer side as the base that it comes as a surprise to some students to find a parallelogram having its shorter side as its base. Emphasize that changing the base from the longer to the shorter side necessitates using a different altitude.

Secs. 187-194 (p. 204). Theorem 17, Corollaries I-VII—By now the system of geometric thought is becoming complex. In discussing the system with students it is probably better, therefore, to show only small sections rather than the whole outline. For example, you can show that Theorem 17 is derived from Theorem 7 and leads to Corollaries I-VII, and later show how corollary I, leads to Theorem 24, which in turn leads to the corollary in § 203, thus



The importance of such work lies not in the particular pattern of relationships, but in student recognition that there is a pattern—that the undefined terms lead to definitions, that definitions combined with assumptions lead to a conclusion, which in turn leads to another conclusion, and so on.

Sec. 195 (p. 205). Theorem 18—Even the weaker students can manage this proof without assistance.

Sec. 196 (p. 205). Theorem 19—Though this proof is easy, weak students will profit from a review of §140 before they begin their work.

Frequently list on the board, "Facts we know about a parallelogram." Such listings help students to organize their thinking.

The exercises on pages 205-206 are valuable. Use as many as time permits.

Sec. 197 (p. 207). Theorem 20—Even average students should be able to demonstrate this theorem without reference to the partial proof in the textbook. Exercise 1 at the bottom of the page is important. Do not neglect it.

Secs. 198-200 (p. 208). Theorems 21-23—Even average students should have no difficulty in demonstrating the proofs of these theorems. Weak students may need to be reminded that in Theorem 22 both pairs of opposite sides must be proved parallel.

Students have a tendency to misquote Theorem 21 by saying, "If the opposite sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram." Point out that while the statement is true, it represents a case of overdetermining information and shows poor thinking.

Sec. 201 (p. 209). Discovering a Proof—It is difficult to set down in a textbook a set of instructions by which students can discover how to proceed in discovering a proof. Either the instructions become so long and complicated that those most in need of the help do not have the patience to read them, or they are so meager that they are of no help. We have included these pages in an attempt to give assistance to students, but the instructions we have written cannot take the place of your day-by-day guidance.

The exercises on page 212 are particularly valuable in encouraging analytical thinking. Use as many as time permits.

Sec. 202 (p. 213). Theorem 24—Make sure that students understand the thought involved in the theorem before they attempt the proof. Make sure, too, that students know what is given and what is to be proved. The proof is not difficult, but it is complicated.

Sec. 203 (p. 213). Corollary—Students sometimes have difficulty in seeing that this corollary is merely an application of the theorem. Probably the greatest student difficulty is in not recognizing that the sides of the triangle are two transversals cutting a set of parallel lines.

Sec. 204 (p. 214). Theorem 25—This theorem is important and merits careful study. The exercises following it help to fix the thought of the theorem. Do not neglect them. Even average students take pleasure in proving the B exercises on page 215.

Sec. 205 (p. 215). Theorem 26)—With the help of the suggestions given in the text, students should have no difficulty in producing a good proof for this theorem. Make sure that students do not later say that the midpoint of the side of any triangle is equidistant from its vertices.

Sec. 206 (p. 215). Trapezoid—Make clear to the class that a trapezoid is never a parallelogram. Make sure, too, that students know how the median of a trapezoid differs from the median of a triangle.

Sec. 207 (p. 216). Theorem 27—Exercises 1-3 at the bottom of page 216 are helpful in emphasizing the thought proved in the theorem. The remaining exercises on pages 216 and 217 are excellent for encouraging students to think analytically.

Secs. 208-211 (p. 218). Theorems 28, 29, and Corollaries—These theorems and corollaries merit careful study. There is a tendency for students to understand them for a few days, then forget them. The exercises of pages 220, 221, and the top of 222 help to fix the concepts proved in the theorems and corollaries. Your better students will enjoy the miscellaneous exercises in Group C. None are too difficult for better than average students.

Sec. 212 (p. 223). Problem 10—Students enjoy this construction and the exercises on page 223. The constructions on pages 224 and 225 are miscellaneous exercises that provide an excellent test of reasoning ability. The range of difficulty in the miscellaneous exercises on pages 225-227 is great. Your best students will enjoy Exercise 15 and will succeed more easily if they prove Exercise 14 first.

Secs. 213-214 (p. 228). Space Geometry—These topics are included for teachers who want to integrate plane and solid geometry.

Sec. 215 (p. 230). Reasoning With False General Statements—This topic is helpful in showing students the value of accepting only general statements which they feel are true.

Chapter 8. Areas of Polygons

Objectives:

To review the concepts "area" and "unit of area."

To teach the derivation and use of formulas for finding the areas of the common polygons.

To review irrational numbers and the solution of quadratic equations.

To indicate the place of approximate number in plane geometry.

Time required: 14-16 days.

Planning the content of various courses:

Basic course—§§225-244, 250-255, 262-263, 271. §§ 245-249, 259-261 depending upon need. §§256-258, 264-270 optional. Students should be held responsible for ability to prove on tests the theorems and corollaries marked with two stars. Theorems and corollaries marked with one star should be proved in class. Unmarked theorems and corollaries may be proved informally and the two problems may be discussed informally. Students should memorize and be able to use the formulas of §§228, 233, 235, 238, 242, 244, 250, 253, and 254. Exercises should include some of A and B exercises of most lists.

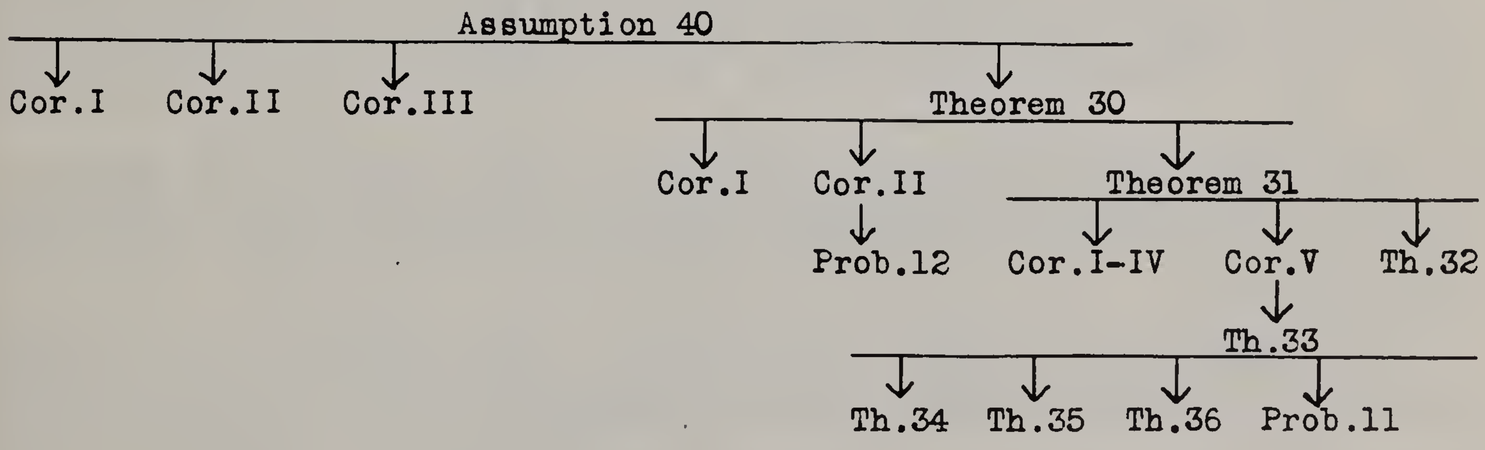
Maximum course for students who plan to specialize in mathematics, science, engineering, and similar fields—§§225-244, 250-258, 262-263, 271. §§245-249, 259-261 depending upon need. §§264-270 optional. Theorems and corollaries should be treated as in the basic course except that students should also be held responsible for deriving the formulas of §§253, 254 and proving Theorem 36. Students should memorize and be able to use the formulas suggested for the basic course and in addition the formula of §257. Exercises should consist of B and C exercises as well as A exercises.

Minimum course—§§225-229, 233-235, 238, 242-244, 250-255, 271. §§245-249, 259-261 depending upon need. Theorems and corollaries should be treated as in the basic course. The exercises should be chosen mostly from A groups, though some of the easier B exercises of page 257 should be included.

Specific suggestions for teaching the various topics:

From time to time you will want to point out how the theorems, corollaries, and problems of this chapter form a small system of logical thought within the larger system. Lead the class to see that the starting point of this small system is Assumption 40.

The following outline shows the relationships of the various theorems, corollaries, and problems. No attempt has been made to trace the lineages to other chapters.



Sec. 225 (p. 237). Measurement of a Surface—Students often arrive at this point in their education without having discovered that in measuring we must use a unit which is like the thing being measured. For example, we measure lengths with small lengths, weights with small weights, volumes with small volumes, angles with small angles, and so on. Make clear that since, in this chapter, we shall be studying areas, the unit must be a small area.

Sec. 226 (p. 237). Unit of Surface—Since the class will need a small area to use as a unit for measuring areas, the logical next question is, "What shall be the shape of the unit of area?"

To help students answer the question draw on the board several rectangles. Cover one with equal circles, one with congruent equilateral triangles, one with congruent parallelograms, one with congruent rectangles, and one with congruent squares, then let the class discuss the merits of the various shapes. Students will soon see that we use a square unit simply because it is the easiest shape to use.

Secs. 227-228 (p. 237). Area of a Figure, Area of a Rectangle—Explain that when we speak of the area of a figure we mean the area enclosed by the figure. Cover a rectangle with small squares as shown in §227 and lead your class to discover Assumption 40. Do not neglect the examples at the top of page 238 since they help puzzled students to recognize that lengths and widths do not need to be expressed as integers or even as rational numbers.

Sec. 229 (p. 238). Ratio—While this topic is not new to most students, this section should be studied carefully since the concept of ratio underlies all of the work in equality and similarity of geometric figures.

Secs. 230-233 (p. 239). Relationships between Rectangles, Corollaries I-III—In courses for average and above average students make sure that the mechanics of proving Corollary I are understood. The method is used again and again in this chapter.

Have a drawing or drawings made for Exercise 4, middle of page 240. The visual impression left by the sketch helps to prevent such errors as $(a + b)^2 = a^2 + b^2$.

Sec. 234 (p. 240). Figures Equal in Area—Make sure that students understand the difference between congruency and equality (equivalency).

Secs. 235-237 (p. 241). Theorem 30, Corollaries I, II—Emphasize that Theorem 30 grows out of Assumption 40. Students have little difficulty with the theorem but tend to be afraid of the corollaries. You may need to use class discussion to get thinking started in the right direction. The exercises on page 242 fix concepts.

Secs. 238-243 (p. 243). Theorem 31, Corollaries I-V—It comes as a revelation to most students to discover that the 1/2 in the formula for the area of a triangle appears because the triangle is one half of a parallelogram. Again you may need to use class discussion to start student thinking in the right direction for proving the corollaries. Use as many of the exercises on page 244 as time permits.

Sec. 244 (p. 245). Theorem 32—Emphasize that this proof is derived from Theorem 31, which is derived from Theorem 30, which is derived from Assumption 40. Choose representative exercises from the list on pages 245 and 246 for class study.

Secs. 245-249 (p. 247). Radicals, How to Find the Square Root of a Number, Quadratic Equations, Incomplete Quadratic Equations, Complete Quadratic Equations—Do not assume that because students have studied these topics previously, there is no need to review them. Most students will profit from review. It is impossible for students to handle the relationships between the hypotenuse and legs of a right triangle, the altitude and sides of an equilateral triangle, the diagonal and sides of a square, and other relationships unless they are familiar with the algebra of these pages. Later work on similarity, regular polygons and the circle, and other topics also make it imperative that students be familiar with the topics discussed in these sections.

Secs. 250-251 (p. 252). The Pythagorean Theorem, Proofs of the Pythagorean Theorem—It has been said that this is the most important theorem in geometry. It is long and complicated, but not difficult for those who analyze and organize their thinking before they begin setting down steps of the proof. Follow the proof of the theorem by enough of the exercises on pages 253-256 to enable students to handle both the thought of the theorem and the mechanics of manipulating the numbers and algebraic relationships with reasonable skill.

Secs. 252-254 (p. 256). Important Formulas, Theorems 34 and 35—Students should study until they are able to derive all these formulas. Following the derivation (not before), the formulas should be memorized. Use enough of the exercises on page 257 to give skill in using Theorems 34 and 35. The formulas given in these theorems are important.

Sec. 255 (p. 258). Theorem 36—Though students intending to specialize in mathematics should be able to prove this theorem, it will be sufficient in most classes to discuss the theorem carefully as a group project. Of course it is better to force individual thinking than to merely have students follow the proof of the text. To force thinking you might have students keep their textbooks closed during the discussion. Draw on the chalkboard a triangle ABC which meets the requirements of the "Given," then lead the class to see that by drawing a second triangle A'B'C' which is a right triangle and which can be proved congruent to triangle ABC, it will be possible to prove triangle ABC a right triangle. Discuss what qualities triangle A'B'C' should have to make it congruent to triangle ABC. Skillful questioning will lead to the discovery that since triangle A'B'C' is a right triangle, the sum of the squares upon its legs is equal to the square upon its hypotenuse, a relationship that seemingly holds for triangle ABC except that we cannot call the sides of triangle ABC its hypotenuse and legs. Students will discover, then, that if A'C' is made equal to AC and B'C' equal to BC, the statements just mentioned will have equal members, thus making the remaining members equal and causing the triangles to have three sides of one equal respectively to three sides of the other.

Secs. 256-258 (p. 259). Hero's Formula, Theorem 37, Corollary—You may feel that for students other than those who will continue to study mathematics, you cannot spare class time for the study of these topics. Even average students enjoy the topics, however, and certainly they should be studied by students who will later specialize in mathematics, engineering, science, and other such fields. The exercises are so constructed that they do not involve difficult computations with fractions, thus allowing students to concentrate on the thought of the theorem without distraction.

Sec. 259 (p. 259). Computation with Approximate Numbers—These pages attempt to clarify the position of approximate numbers in geometry and to outline procedures for using such numbers. Most of the exercises of the textbook involve only exact numbers in order to keep the attention of students on the task of learning to reason deductively. However, since in the geometric situations of real life, most of the numbers we use are the result of measurement, and therefore approximate, students should have some acquaintance with operations involving these numbers. The amount of study that students will need to give to these pages will depend upon their past experience with approximate computation.

In the exercises on page 262 all numbers should be considered approximate. Note that since the measurements in Exercises 6, 7, 8, and 10 have been made only to the nearest foot, the solutions are quite inconclusive. This will provide opportunity for you to point out that if more precise results are desired, the original measurements must be made more precisely.

Secs. 260-261 (p. 263). Area of Polygons, The Trapezoid Formula—In the exercises of these sections students are either told which numbers are to be considered approximate, or are told that they are the result of measurement. Since it was pointed out in Chapter 1 and in Section 259 that only approximate measurements can be made, students should infer that when a number is said to be the result of measurement it should be considered approximate.

In Exercise 1 of page 265 we consider that all measurements have been made to the nearest foot. In Exercise 2 of that page, since the least precise measurement has been made to an inch, the more precise measurements are wasted. All measurements should be rounded off to the nearest inch.

Sec. 262 (p. 267). Problem 11—Many students feel that they understand Problem 11, yet they are unable to use the problem in the exercises at the bottom of page 267. Make sure that what your students call "understanding" is not mere memorization.

Sec. 263 (p. 268). Problem 12—Since students are sometimes confused by all the auxiliary lines in the figure of Problem 12, it is better to discuss the construction with a new polygon drawn on the board—one that shows no auxiliary lines at the start. Then by drawing one diagonal that cuts off a triangle, together with the other lines that must be used to show that the triangle cut off is equal to the new triangle being added to the figure, students can follow the procedure as it is carried out. While you should usually require that all arcs and auxiliary lines be left in a drawing, in this case it is helpful to erase all but those in use at a given time. By repeatedly cutting off triangles until the original polygon is reduced to a triangle, students can follow the construction without difficulty.

Secs. 264-270 (p. 274). Space Geometry—These topics are included for teachers who want to integrate plane and solid geometry.

Sec. 271 (p. 278). Determining, Overdetermining, and Underdetermining—Since these paragraphs point out some very common weaknesses in reasoning, the topic should be studied by all students.

Chapter 9. Circles, Angles, and Arcs

Objective:

To extend the system of geometric thought to include the relationships between circles and their angles and arcs.

Time required: 7-9 days.

Planning the content of various courses:

Basic course—§§276-288, 290-313, 307. §§289, 304-306 optional.

Students should be held responsible for ability to prove theorems marked with two stars, except that Theorems 44 and 45 and their corollaries may be accepted without proof if desired. Theorems marked with one star should be proved in class. Unmarked theorems may be proved in class informally. Students should study exercises chosen from both the A and B groups.

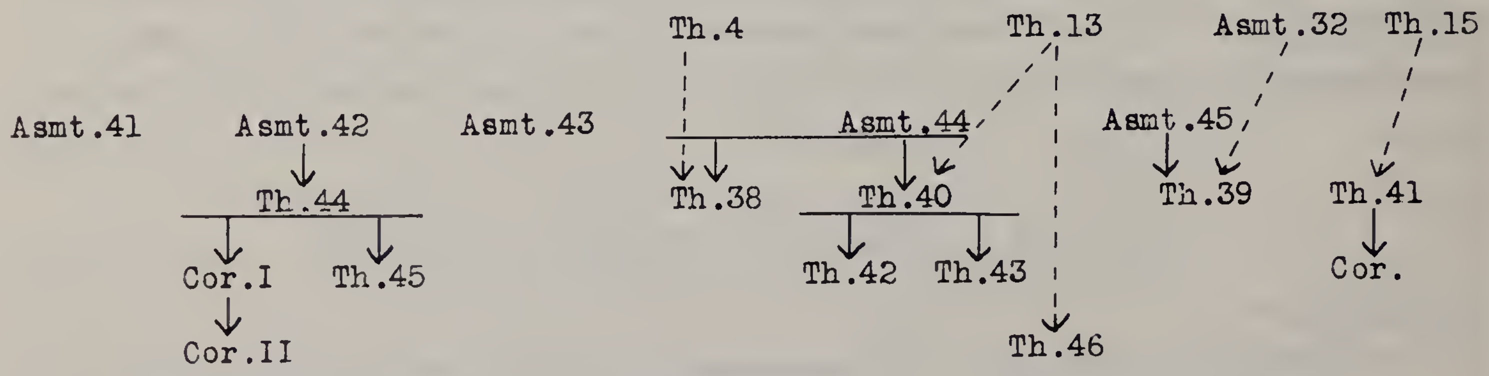
Maximum course for students who plan to specialize in mathematics, science, engineering, and similar fields—§§276-288, 290-303, 307. §§289, 304-306 optional. Students should be held responsible for ability to prove on tests the theorems marked with two stars. Theorems marked with one star should be carefully proved in class. Unmarked theorems and corollaries may be proved in class informally.

Minimum course—§§276-283, 290-303, 307. §289 optional. Theorems should be treated as suggested for the basic course except that Theorems 38 and 39 may be accepted without proof, if desired. Exercises will necessarily be confined mostly to A groups.

Specific suggestions for teaching the various topics:

Point out to students that since the study of circles is to be a part of the system of logical geometric thought they are building, they must continue to use the same undefined terms, definitions, assumptions, and proved theorems and corollaries that they have been using. These are not sufficient, however, to allow much thinking about circles. Additional definitions and additional assumptions specifically related to circles are needed. §§ 276-280 present additional definitions and §§281-284 present additional assumptions.

From time to time show students how the system of thought is developing. In the outline below the dotted lines show relationships to major theorems of preceding chapters, but no attempt has been made to show all such relationships. Neither does the outline show dependence upon definitions, and assumptions.



Sec. 276 (p. 285). Circle—If you prefer, you may define a circle by saying: A circle is the set of points in a plane which are equidistant from a point of the plane called the center. Whether you use this definition or the one in the text, notice that the circle does not include the portion of the plane enclosed by the curve. Technically, the center is not a point of the circle.

Secs. 277-280 (p. 285). Lines Related to the Circle, Angles Related to the Circle, Arc, Circumference—Students should study the definitions of all the italicized words. Keep reminding your class that definitions are bases of proof.

Secs. 281-285 (p. 286). Assumptions Related to a Circle, Assumption 44, Corollary, Assumption 45, Corollary—Emphasize meanings of these assumptions through drawings and discussion. You will recognize that some of these assumptions are unproved theorems. Since the proofs are difficult and time-consuming, it seems wiser to list the statements as assumptions.

Secs. 286-287 (p. 290). Theorems 38, 39—In some classes you may wish to have students accept Theorems 38 and 39 as true without proof. College-bound students should study the proofs.

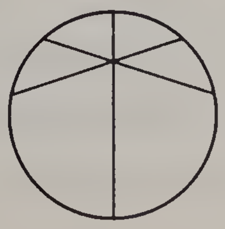
Sec. 288 (p. 290). Inscribed Polygons—Make sure students can correctly use the italicized words of this paragraph. Give special thought to Exercise 6 on page 291 since many students fail to handle it properly. Exercises 8-10 are not too difficult for your average students.

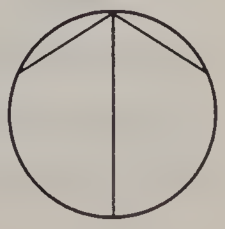
Sec. 289 (p. 292). Geometry in the Arts—This section contains enrichment material for students interested in art.

Secs. 290-292 (p. 295). Theorems 40, 41, Corollary—Students find these theorems both easy to prove and easy to remember. The corollary is more difficult to prove, but easy to remember. Use as many of the exercises on page 296 as time permits.

Sec. 293 (p. 297). Theorem 42—An interesting project related to this theorem is done as follows: Draw a large circle and divide it into equal parts (from 24 to 48 parts give the best results unless the circle is very large). Draw all the possible chords joining these points of division. Each set of equal chords then appears to form a circle as in Exercise 1 of page 297, and the set of all chords appears to form concentric circles. By using thread and making some of the chords of one color and others of other colors very beautiful designs can result.

Sec. 294 (p. 298). Theorem 43—Expect students to use either of the drawings at the right for Exercise 2. Probably the first drawing more nearly meets the requirements of the exercise since we are told that the chords "intersect" the diameter. In a





strict sense the chords of the second drawing merely meet the diameter. However, you will probably want to accept either interpretation.

Students are usually surprised to find that Exercise 3 is somewhat more difficult than it appears to be.

Secs. 295-298 (p. 299). Theorem 44, Corollaries I, II, Theorem 45—Students who intend to specialize in mathematics or related fields should prove these statements either through group discussion or through their individual efforts. However, in classes for other students you may want these statements accepted as true without proof.

If you use the proofs, make clear that since AB is a tangent to the circle, every point on AB (except C) lies outside the circle. Since it is impossible to deal with all of these outside points, we choose one to represent the group, making sure that what we say about it would be equally true of all the others.

Since the four statements in this group of theorems and corollaries are so much alike, it is wise, after the proofs are completed, to take a second look at the statements noticing their similarities and differences.

Sec. 299 (p. 300). Tangent and Secant from an External Point—Notice that while a tangent extends infinitely far in two directions from the point of contact, this definition is concerned only with the distance from an external point on the tangent to the point of contact. Similarly, though a secant extends infinitely far in two directions from either point of intersection with the circle, this definition is concerned only with the distance from an external point on the secant to the more remote of the two points of intersection. These definitions are introduced to make it easier to state Theorems 46 and 71.

Secs. 300-301 (p. 301). Theorem 46, Theorem 47—Even average students should have no difficulty with the proofs of these theorems. Use enough of the exercises on pages 301-305 to give your students experience in working with tangents and chords. You probably will also want to use a few of the exercises on pages 306-307.

Secs. 302-303 (p. 308). Tangent Circles, Common Tangents—Make sure that students understand and can use the definitions of this section. They should study as many of the exercises on pages 309-312 as time permits.

Secs. 304-306 (p. 312). Space Geometry—These topics are included for teachers who want to integrate plane and solid geometry.

Sec. 307 (p. 314). Assuming the Converse in Everyday Reasoning—Even though this topic is optional, as many students as possible should study it.

Chapter 10. Measurement of Angles and Arcs

Objective:

To extend the system of geometric reasoning to include the measurement of angles and arcs.

Time required: 5-7 days.

Planning the content of various courses:

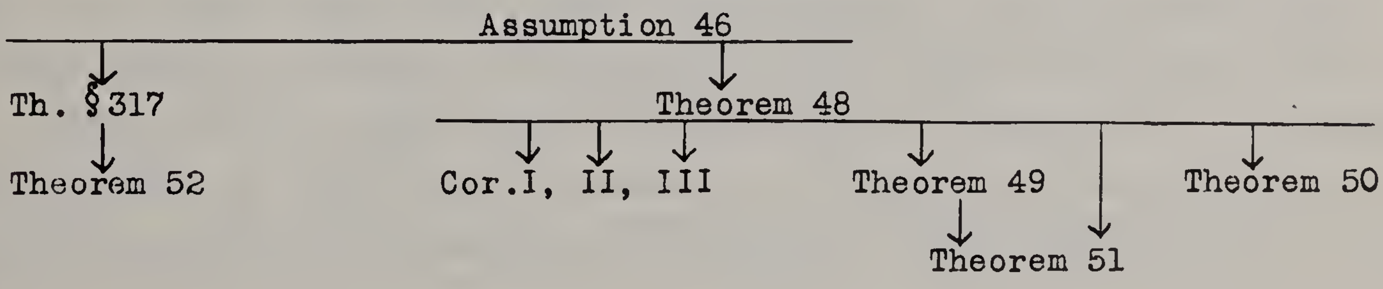
Basic—§§314-332. Students should be able to prove the theorems marked with either one or two stars and to carry out the steps of the construction and proof for each problem marked with two stars. Unmarked theorems should be completely proved in class. The unmarked problem may be discussed informally.

Maximum course for students who plan to specialize in mathematics, science, engineering, and similar fields—Same as basic course except that more attention should be given to B and C exercises.

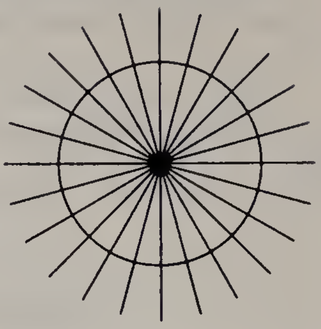
Minimum course—Same as basic course except that exercises will be confined mostly to A groups.

Specific suggestions for teaching the various topics:

This chapter adds a new assumption and from it extends the system of deductive reasoning to include a set of new geometric relationships concerning the measurement of angles and arcs. The following outline shows the relationships of this chapter. No attempt has been made to trace the relationships to previous chapters.



Sec. 314 (p. 321). The Central Angle and Its Arc—Make sure that students understand the distinction between an angle degree and an arc degree. You may help students to visualize the situation by drawing a circle O and showing that if we divide the space around O into 360 small angles we call each a degree. Since these degrees are small angles they are called angle degrees. Show that the sides of the angles intersect the circle forming small arcs which are called arc degrees.



Sec. 315 (p. 321). Assumption 46—Point out that to say that a central angle equals its arc is as foolish as to say 4 tablespoons equal 4 miles. It is true that both the tablespoons and the miles are expressed as 4 units, but the resemblance ends there because we cannot compare unlike quantities such as tablespoons and miles. Both the central angle and the arc may contain the same number of units (for example in the illustration of the text both contain 50 units) but the units are unlike quantities—one is an angle and the other is an arc. Emphasize that the assumption really expresses the thought that an arc contains the same number of arc degrees as its central angle contains angle degrees.

Sec. 316 (p. 321). Comparison of Arcs—You may help to explain that two arcs can contain the same number of degrees and yet be of different lengths by having the class assume that both arc AB and arc DC of the figure in the text contain 60°. Then since a whole circle contains 360°, arc AB = $\frac{60}{360}$ or $\frac{1}{6}$ of the circle of which it is a part, and arc CD = $\frac{60}{360}$ or $\frac{1}{6}$ of the circle of which it is a part. Obviously, $\frac{1}{6}$ of the smaller circle is not equal to $\frac{1}{6}$ of the larger circle.

Sec. 317 (p. 322). Theorem—This theorem and the exercises following it should cause no difficulty.

Sec. 318 (p. 324). Theorem 48—There are three cases for this theorem because the angle can be placed in three positions with respect to the center of the circle. Since each of the three positions requires different treatment, there must be three proofs. Case I is the basic case. The other two proofs can easily be deduced from it.

Secs. 319-321 (p. 325). Corollaries—Students enjoy these corollaries and find them easy to use. The class should study as many of the exercises as time permits.

Sec. 322 (p. 326). Theorem 49—The proof of the theorem should cause no trouble. In Exercise 7 on page 327 students sometimes hesitate over the statement that arc BD $\overset{\circ}{=}$ arc BC. If so, refer them to §316. Exercises 10-12 are not too difficult for your average students. An occasional opportunity to do a C exercise strengthens morale.

Sec. 323 (p. 328). Theorem 50—Students usually like this theorem and the exercises following it. They usually consider Exercise 13 more difficult than it really is, and after they have discovered the proof wonder why they were so slow to find it.

Secs. 324-325 (p. 330). Theorem 51, Circumscribed Polygon—Note that the figures have been so lettered that the same proof can be used for all cases with one exception—the reasons for statements 4 and 5 will vary according to the figure under consideration. The class should study as many of the exercises on page 331 as possible.

Sec. 326 (p. 332). The Principle of Continuity—One characteristic of today's mathematics is the grouping of related bits of information. By recognizing that the topics in §§315, 318, 322, 323, and 324 are all aspects of one larger topic, we have only one thought to remember instead of five separate thoughts.

Sec. 327 (p. 334). Theorem 52—To provide further practice after completion of Theorem 52, prove (1) Theorem 49 using an auxiliary line through C and parallel to AB in the figure on page 326, (2) Theorem 50 by drawing a line through A and parallel to DC in the figure on page 328, and (3) Theorem 51 by drawing a line through C and parallel to AB in the figures of §324 on page 330. These proofs are entirely possible since Theorem 52 could have been proved before Theorems 49-51.

Secs. 328-330 (p. 335). Problems 13, 14, 15—Problem 15 is the only one of these problems which causes much difficulty. Students have a tendency to do this one mechanically with no regard as to why the various steps are used. Consequently, they quickly forget the procedure. Have students keep books closed as they first make the construction. Use plenty of follow-up exercises to strengthen the concepts involved.

Secs. 331, 332 (p. 336). Problems 16, 17—These problems can become the basis for exercises in making designs. With a little encouragement some very beautiful ones can result.

Do not overlook the reasoning exercises on page 340.

Chapter 11. Loci

Objective:

To understand and be able to use the locus theorems listed on page 378.

Time required: 8-13 days.

Planning the content of various courses:

Basic course—§§338-348, 350, 355-371. §§372-375 if possible. §§349, 351-354, 376, 377 optional. Emphasis should be upon understanding of the theorems and ability to use them rather than upon ability to prove them. Theorems and corollaries marked with one or two stars should be completely proved in class. Unmarked theorems and corollaries may be proved informally in class. Problems marked with either one or two stars should be made and proved in class. The unmarked problem of page 368 should be done if time permits. Students should do the A exercises and some of the B exercises of the topics listed.

Maximum course for students who plan to specialize in mathematics, science, engineering, and similar fields—§§338-375. §§ 376, 377 optional. Emphasis should be upon understanding of and ability to use theorems rather than upon ability to prove them, but it is hoped that students in this course will be able to prove Theorems 53 and 54 and to informally explain the others. They should be able to make the constructions and give proofs for Problems 18-20. While proofs should not be required for all exercises, these students should prove enough representative exercises to learn the procedure.

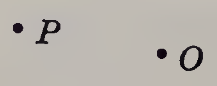
Minimum course—§§338-348, 350, 355-371. §§349, 351-353 optional. Students should be able to understand and use the theorems in the sections suggested for this course. These theorems and corollaries should be proved in class. Problems 18, 19 should be done in class. Exercises should be confined mostly to the A exercises in the sections suggested for study.

Specific suggestions for teaching the various topics:

This chapter requires careful teaching. With good guidance students enjoy the topic and profit greatly from its study. With poor guidance they can become hopelessly confused.

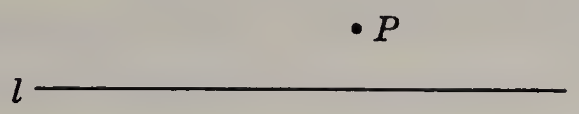
Sec. 338 (p. 347). Locus of Points—Comment that a locus is the path of a point that moves so as to constantly fulfill certain conditions, then illustrate by examples of this type:

1. O is a fixed point. P is a movable point 2 inches from O.

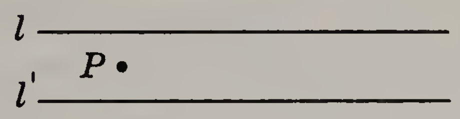


Suppose that I let point P move and impose upon it the condition that it always remain 2 inches from O, what will be its path?

2. P is a movable point one inch above a fixed line ℓ. Suppose that I let point P move and impose upon it the condition that it always remain 1 inch above ℓ, what will be its path (locus)?

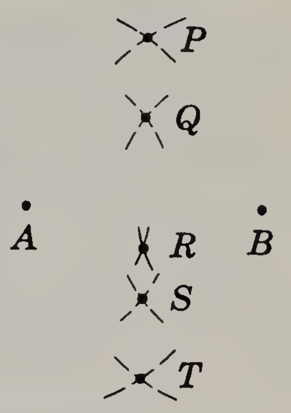


3. ℓ and ℓ' are parallel lines. P is a point midway between the lines. Suppose I let point P move and impose upon it the condition that it always stay as close to ℓ as to ℓ'. What will be its path (locus)?

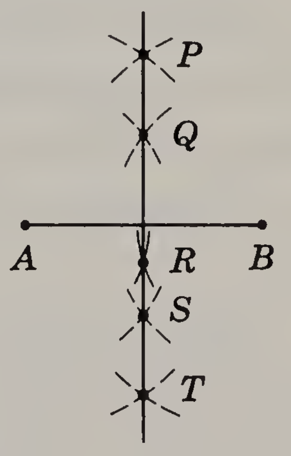


A locus can be defined as the set of points, and no others, that satisfy a given condition or set of conditions.

Sec. 339 (p. 348). Directions for Determining a Locus—Explain that in the exercises just described the locus was easy to determine, but that in some exercises determining the locus requires more careful analysis. For example, if we were asked to find the locus of points equally distant from two fixed points A and B, we might proceed by first finding several points that satisfy the given condition (Step 1 of the Directions) as in the drawing at the right:



Then we might follow the instructions of Step 2 of the Directions and draw a smooth line between these points (P, Q, R, and S and T in the drawing), as at the right: Next we might try to describe the line (or other figure) which we have drawn (Step 3 of the Directions). In our drawing the line appears to be the perpendicular bisector of AB.



Finally, we would prove that the perpendicular bisector of AB is the locus (Step 4). More will be said about this step in §342.

Sec. 340 (p. 348). Distance from a Point to a Circle—Students have a tendency to want to speak of any line segment from a point to a circle as the distance from the point to the circle. You may need to remind them that by distance we mean the distance measured along the line joining the point to the center of the circle.

Notice that the exercises on pages 348 and 349 omit Step 4 of the Directions for Determining a Locus.

Sec. 341 (p. 349). Assumptions on Loci—Call attention to the fact that these assumptions (really unproved theorems) allow us to enlarge our system of geometric thought to include theorems on loci. If students have doubts about these assumptions use methods similar to those suggested for § 338.

Sec. 342 (p. 350). Proving a Locus Theorem—Notice that this topic gives an explanation for Step 4 of the Directions for Determining a Locus, on page 348. Emphasize and review that in proving a geometric figure a locus we must give two proofs as indicated in the box on page 351. Point out that these two proofs are the proofs of a statement and its converse. While you may not hold all of your students responsible for being able to prove a locus problem, all students should understand the nature of such proof.

Secs. 343-344 (p. 352). Theorem 53, Corollary—It is suggested on page 352 that proofs are not required. However, you will probably want your better students to write proofs for an exercise or two on this page. Weak students sometimes become discouraged over proofs and give up learning more about loci.

Secs. 345-346 (p. 353). Theorems 54, 55—Theorem 54 and the first part of Theorem 55 are easy enough, especially since these statements have been proved previously, but Part II of Theorem 55 causes trouble for some students. Proceed slowly and carefully here.

Secs. 347-348 (p. 354). Theorem 56, Concentric Circles—With reasonable care this theorem causes little trouble. In the exercises on page 354 use your judgment about requiring proofs. Good students can be expected to write a proof or two, but average and weak students will probably not profit from being required to give them.

Sec. 349 (p. 355). Ways of Expressing a Theorem and Its Converse—Since the expressions "if and only if" and "necessary and sufficient" are so common in every-day language as well as in advanced mathematics courses, try to have your class study this topic.

Sec. 350 (p. 356). Compound Locus—A few students will probably become confused on these, but the exercises give excellent practice in analytic thinking for able students.

Secs. 351-354 (p. 358). Loci Which are Conic Sections, Algebraic Loci, Linkages, and Other Loci—These topics help you to tie geometry to the world in which we live as well as to algebra and to coordinate geometry. Use models of cones which show the conic sections or have students make such models. Have students construct the circle, ellipse, parabola, and hyperbola as locus drawings. Have them make a linkage or two. There are many ways in which these topics can be made lively.

Secs. 355-359 (p. 361). Concurrent Lines, Theorem 57, Circumcenter, Theorem 58, Incenter—Students have little difficulty with §§356 and 358 at the time they are studied. After a week or two, however, students tend to forget whether the perpendicular bisectors produce the circumcenter or the incenter, and there is similar confusion about the angle bisectors. Try to prepare your students for this time by helping them to think in terms of loci. When a student forgets, ask "What do you know about the points on the perpendicular bisector of a line segment?" "Will this fact make the point of concurrency of the perpendicular bisectors equidistant from the sides of the triangle, or from its vertices?"

Secs. 360-361 (p. 364). Theorem 59, Orthocenter—Some students find it difficult to see the parallelograms. In starting this theorem first lead students to analyze the problem and decide upon a procedure. Move through the proof a section at a time, making sure that each step is thoroughly understood before a second is begun. Such questions as the following may serve as a guide: "How can we prove CF the perpendicular bisector of A'B'?" "Who will prove that CF is perpendicular to A'B'?" "Who will prove that CF bisects A'B'?" Repeat for AD and EB. Finally, ask what we know about the perpendicular bisectors of the sides of a triangle.

Secs. 362-363 (p. 365). Theorem 60, Centroid—Many teachers find they can teach this theorem best by first drawing only two medians (with perhaps the rudiments of the third to indicate that eventually a third will be considered). For example, using the figure on page 365, these teachers would use only AD and BE and proceed through the proof according to Statements 1-10. At this point AD would be erased, although O, its intersection with EB, would be kept. Now by drawing CF and letting O' be its intersection with EB the procedures of Statements 1-10 would be repeated. This produces the statement: BE and CF intersect in a point O' which lies 2/3 of the way from B to E and from C to F. It would then be pointed out that since this statement says that BO' is 2/3 of BE and since the corresponding statement for the first pair of medians states that BO = 2/3 of BE, then BO' = BO, whence O and O' coincide, and CF passes through O.

Probably one of your students would be interested in cutting a triangle from stiff cardboard or metal, then locating the centroid, and showing how the triangle will balance on the tip of a pencil when the centroid is at the tip.

Secs. 364-370 (p. 366). Problem 18, Circumcircle, Corollaries I, II, III, Problem 19, Incircle—As has been stated previously, students have a tendency to forget whether the perpendicular bisectors or the angle bisectors produce the circumcenter. Similarly, for the incenter. Keep your class thinking in terms of loci.

Sec. 371 (p. 367). Segment of a Circle—Note that a segment of a circle is defined, not as a portion of a plane, but as a combination of two lines--an arc of a circle and its chord.

Sec. 372 (p. 368). Problem 20—Students find this construction somewhat complicated. They also tend to overlook using it when they need it.

Sec. 373 (p. 369). Notation in a Triangle—This notation makes the task of discussing constructions both easier and more compact.

Sec. 374 (p. 369). Analysis of a Problem--You cannot overemphasize the need for taking the time to draw a sketch and to think through a procedure before starting to make a construction. Many students want to jump into the actual work of drawing before they have a plan.

Sec. 375 (p. 369). Loci in Construction—Experience plus reminders from you will teach students the value of thinking in terms of loci when doing these construction problems. Your better students should, if possible, spend three or four days on pages 371-374. The problems are excellent for developing the power to think analytically.

Sec. 376 (p. 375). Space Geometry—This section is included for teachers who want to integrate plane and solid geometry.

Sec. 377 (p. 376). Indirect Reasoning—These exercises make good motivation material for use at any appropriate time, though preferably not before students have studied indirect proof.

Chapter 12. Proportion, Proportional Line Segments

Objectives:

To extend the system of deductive geometric thought to include the concepts of proportion and proportional line segments.

To increase ability in using proportions.

Time required: 6-8 days.

Planning the content of various courses:

Basic course—§§381-393, 395-403, 405. §§394, 404, 406, 407, 408 optional. The fundamental theorems on proportion should be proved informally. Theorems 61 and 62 may either be accepted as true or proved informally. Theorem 63 should be proved in class but the proof may be more or less informal. If Theorem 64 is proved, §404 should be studied prior to the proof. Students should be held responsible for giving the constructions and proofs for Problems 21 and 22. Exercises should be chosen from A and B groups.

Maximum course for students who plan to specialize in mathematics, science, engineering, and similar fields—§§381-408. Students should be able to give informal proofs for the fundamental theorems on proportion. They should be able to prove Theorems 63 and 64 and to explain informally Theorems 61 and 62. Corollaries should be proved in class, but only those marked with a star need have formal proofs. Exercises should be chosen from C groups as well as from A and B groups.

Minimum course—§§381-393, 395-403. Other topics optional. The fundamental theorems on proportion should be proved informally. Theorems 61 and 62 may be accepted as true without proof. Theorem 63 and all the corollaries may be proved informally. Students should be held responsible for ability to make the constructions and give proofs for Problems 21 and 22. Exercises should consist mostly of A exercises.

Specific suggestions for teaching the various topics:

Sec. 381 (p. 381). Ratios—Not all textbooks adhere to the plan of writing a ratio as a fraction. For example, in cases such as $\frac{3}{1}$ some texts write only 3. This text uses the more complete form $\frac{3}{1}$ with the thought that it expresses the idea of relationship better than the 3 alone.

Secs. 382-385 (p. 381). Proportion, Terms of a Proportion, Fourth Proportional, Mean Proportional and Third Proportional—Make sure that students learn the meanings of the italicized words and use them properly. Students should be able to do all the types of exercises on page 382 before proceeding further.

Secs. 386-393 (p. 383). Fundamental theorems on proportion—Students should know the thoughts involved in these theorems and should be able to use the theorems. Good students should be able to prove them.

Sec. 394 (p. 386). Commensurable and Incommensurable Quantities—Use Exercise 3 on page 387 to further develop the idea of incommensurable quantities. Let the students find $\sqrt{2} = 1.4142+$. Bring out the fact that no matter how many decimals are used there is always a remainder. Now tell the class that you want to find a unit of measure that will be contained in both the side and the diagonal an integral number of times. Ask whether the unit .1 inch will meet the requirements. Discussion should bring out the fact that while .1 inch will be contained in the side 10 times it will not be contained in the hypotenuse an integral number of times—the quotient will be 14 with a remainder of .142+. Now suggest .01 as a unit and let discussion show that it is equally unsuitable. Continue until the class recognizes that no matter how small the unit, there will always be a remainder when the hypotenuse is divided by the unit. Now bring out the fact that if an irrational number is used as a unit, one that would be contained in the hypotenuse an integral number of times would not be contained in the side an integral number of times. For example, if $\sqrt{2}$ inches were used as a unit it would be contained in the hypotenuse 1 time but in the side .7071+ times.

Secs. 395-396 (p. 386). Ratio of Geometric Quantities, Line Segments in Proportion—Exercises 7-9 of page 387 will help you further develop the meaning of "two line segments divided proportionally."

Secs. 397-399 (p. 388). Theorem 61, Corollaries I, II—To make clear that the particular numbers 3 and 2 do not affect the proof, have students demonstrate the proof using other numbers. Point out that the ratio $\frac{3}{2}$ disappears from consideration in Statement 6.

Secs. 400-401 (p. 390). Problems 21, 22—These constructions cause little difficulty, but need to be followed by plenty of practice on exercises like those on pages 390 and 391.

Secs. 402-403 (p. 392). Theorem 62, Corollary—Make sure that you know how to handle the proof of Theorem 62 before you begin presenting it to the class. It is not difficult, but does require special handling.

Sec. 404 (p. 393). Internal and External Division of a Line Segment—Explain that in both internal and external division three segments are involved—the original segment, the portion reaching from one end of the original segment to the point of division, and the portion extending from the point of division on to the other end of the segment. If students remember that even in external division the two portions extend from the point of division to the two ends of the segment, they avoid the common error of reading the original segment as one of the portions. Sometimes students are helped by being told that though up to this time they have not had to consider line segments as positive or negative, in this situation direction is involved. Point out that in the "To prove" of Theorem 64 on page 395, AD is read from left to right while DB is read from right to left. If AD represents a positive segment then DB represents a negative segment, then AD + DB really becomes AD - BD, or AB.

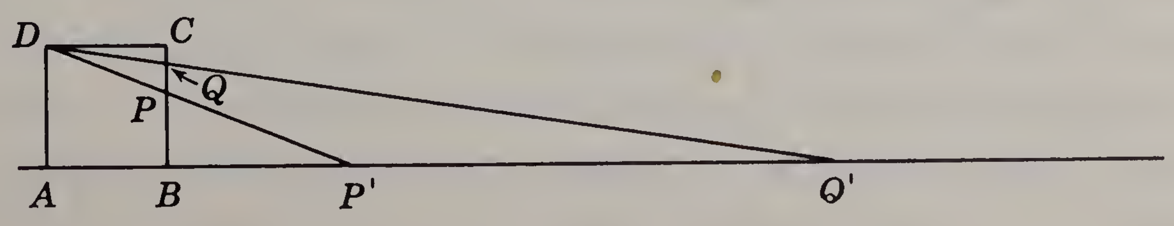
Sec. 405 (p. 394). Theorem 63—Allow enough time on this proof for students to assimilate the thoughts involved. They like the theorem when they understand it. Be sure to have them study at least Exercises 1 and 2 following the theorem. Prove Exercise 3, if time allows. Exercises 3, 4, 7, 8, and 9 of page 396 also help to fix the thought involved in this theorem.

Sec. 406 (p. 395). Theorem 64—Again allow plenty of time for students to assimilate the ideas involved. If you have a strong class that enjoys philosophical considerations ask them what happens if AC = BC. Remind them that in projective geometry we say that parallel lines meet at infinity, therefore according to that way of looking at the results of Theorem 64, we have AD = BD. Point out that according to our finite way of looking at the matter BD is only a part of AD, consequently we have an example in which the whole is equal to one of its parts.

Strong classes profit from further consideration of infinite sets in which a one-to-one correspondence can be set up as in the set of all positive integers and the set of positive even integers. Here again students are amazed to see that though the set of positive even integers is only a part of the set of all positive integers, each integer of one set can be matched by an integer of the other.

1	2	3	4	5	6	7	8	9	10	11	. . .
2	4	6	8	10	12	14	16	18	20	22	. . .

Another problem enjoyed by these students is the one in which a very small square rests upon an infinite line in such a way that a one-to-one correspondence can be set up between the points on the side of the square and the infinitely long line upon which it rests, by projecting the points from the opposite vertex as shown. Students



are interested to find that there are as many points on the side of the square as upon the whole line, provided C corresponds to the infinitely distant point.

Sec. 407 (p. 395). Harmonic Division of a Line Segment—Students may be interested in doing further reading about Pythagoras and his discovery of harmonic division.

Sec. 408 (p. 397). The Use of Analysis in Algebraic Proofs—This section should be studied by all students who intend to continue the study of mathematics.

Chapter 13. Similar Polygons

Objective:

To extend the system of geometric thought to include the concept of similarity.

Time required: 12-16 days.

Planning the content of various courses:

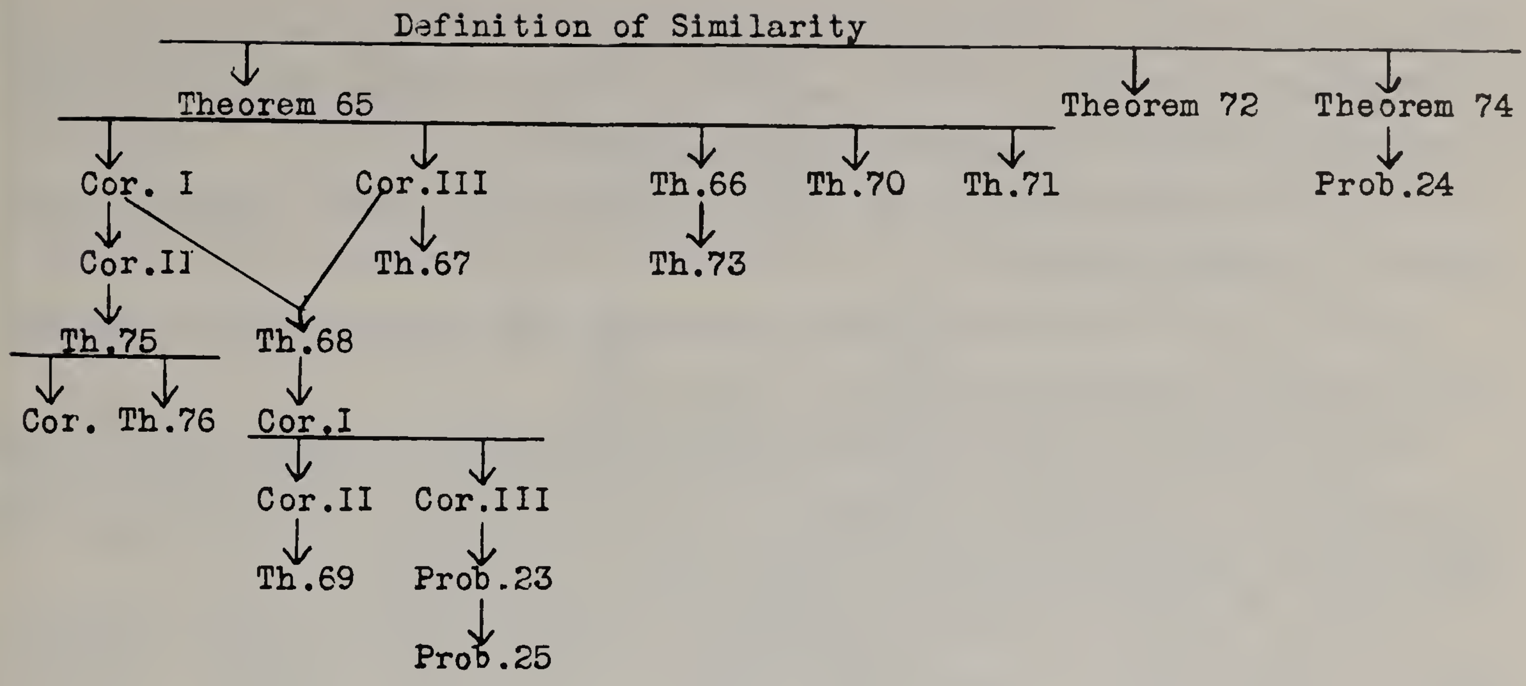
Basic course—§§412-443. §§444-449 optional. Students should be held responsible for ability to prove the theorems and corollaries marked with two stars, except that Theorem 76 may be proved informally, or even accepted without proof. Theorems and corollaries marked with one star should be proved in class (Theorem 73 may be accepted without proof). Unmarked theorems and corollaries may be proved informally. Students should be held responsible for ability to construct and prove Problem 23. Problems 24 and 25 may be done informally. Students should do as many A and B exercises as time permits.

Maximum course for students who intend to specialize in mathematics, science, engineering, and similar fields—§§412-446. §§447-449 optional. Theorems should be treated as suggested for the basic course, but exercises should be chosen from B and C groups as well as from A groups.

Minimum course—§§412-443. Students should be able to prove the theorems and corollaries marked with two stars except that Theorem 76 may be accepted without proof. Theorems and corollaries marked with one star should be proved in class although Theorems 67, 73, and 74 may be accepted without proof. Unmarked theorems may be proved informally. Exercises will need to be confined mostly to A groups.

Specific suggestions for teaching the various topics:

In this chapter we extend our system of geometric thought through the definition of similarity. The outline at the top of the next page shows the sequence of the theorems of this chapter. No attempt has been made to trace their lineage to previous chapters.



Sec. 412 (p. 403). Similar Polygons—When beginning this chapter, students should understand that there are two requirements for polygons to be similar.

They should realize that the word "corresponding" now takes on a more general meaning. Heretofore the word "corresponding" has been associated with the symbols = and ∼ combined into the symbol ≅. From now on the word "corresponding" will be associated with the symbol ∼ regardless of the = sign.

Sec. 413 (p. 403). Conditions of Similarity—Use this section to emphasize the definition of similar polygons.

Sec. 414 (p. 404). Ratio of Similitude—The exercises on page 404 are applications of similarity and proportional line segments.

Secs. 415-416 (p. 405). Theorem 65, Corollary I—Some texts state §416 as a theorem and Theorem 65 as a corollary.

If desired, Theorem 65 can be proved without using superposition. In this case Statement 4 of the proof would be, "Construct △CA'B' ≅ △A'B'C' (§169)." Then statement 6 would need to be modified.

Secs. 417-418 (p. 406). Corollary II, Corollary III—Students should be able to prove these corollaries if they follow the suggestions.

Sec. 420 (p. 406). Methods of Proving Line Segments Proportional—Allow sufficient time to discuss this section with the class. If the students have a clear understanding of this section, they will have the proper "tools" and know which "tool" is needed when they are discovering the proof of an original theorem.

Sec. 421 (p. 408). Ways of Proving Products of Line Segments Equal—Students should learn to associate products of line segments with proportions involving line segments. They should understand that equal products of line segments often can be obtained from proportions whose terms are line segments.

The two common types of equal products of line segments are listed in this section.

Note that exercises 8 and 9 on page 409 are skeleton proofs of theorems appearing later in the text.

Sec. 422 (p. 410). Theorem 66—Theorem 66 is used quite often in proofs of original theorems. The theorem can be proved without the use of superposition but the proof is longer.

When proving the theorem without using superposition, Statement 1 would be "1. Construct $\triangle CA'B' = \triangle C'A'B'$ (§169)." Then the remainder of the proof would need to be modified in some places.

Sec. 424 (p. 413). Theorem 68—Theorem 68 and Corollary I are important. Their proofs are easy.

Sec. 426 (p. 414). Projection of a Line Segment—If desired, instead of defining the projection of a line segment on a line as is done in the text, you may define the projection of a point on a line. In this case it will be necessary to prove that the locus of the projections of the points of AB is A'B'.

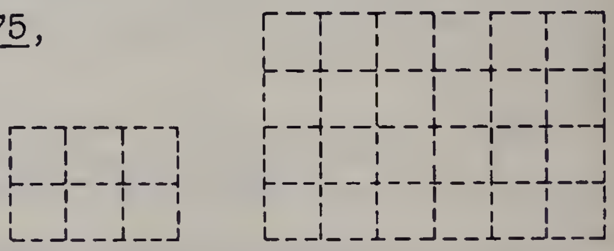
Secs. 427-428 (p. 414). Corollary II, Corollary III—Students who forget these two corollaries can often use the theorem as a substitute.

Sec. 430 (p. 417). Theorem 69—This proof of the Pythagorean theorem is the one usually preferred.

Sec. 432 (p. 418). Segment of a Secant—This section explains the common usage of "secant."

Sec. 434 (p. 422). Theorem 72—Ask students if Theorem 72 can be proved before proving Theorem 65.

Secs. 435-436 (p. 423). Theorems 73, 74—Theorem 73 has more uses than Theorem 74.



Secs. 438, 439 (p. 427). Theorem 75, Corollary—Before beginning the proof of Theorem 75, a chalkboard drill on the areas of similar rectangles is desirable. Rectangles having ratios of similitude of 1:2, 1:3, 2:3, and 3:4 can be used.

For the ratio of similitude 1:2, a diagram like the one shown here can be used. Then questions like the following may be asked:

What is the ratio of the bases of the two rectangles? What is the ratio of the altitudes of the rectangles? Are the rectangles similar? Why? What is the number of square units of area of each rectangle? What is the ratio of the areas of the rectangles? Is the ratio of the areas equal to the ratio of the bases? Is the ratio of the areas equal to the ratio of the altitudes? Is the ratio of the areas = $\frac{2}{3} \times \frac{2}{3}$? Is it = $\frac{2^2}{3^2}$?

Sec. 441 (p. 430). Problem 25—Students will need little help from the teacher on this problem.

Sec. 442 (p. 430). Solving Problems—Elaborate on this discussion. Some of the problems on page 431 can be used to show how algebra helps to discover the solution.

The 60 exercises on pages 432-436 offer a wide range of selection for assignments. An average class will be able to solve most of the exercises on page 432 in one assignment.

Select exercises from pages 433-436 for the better-than-average students.

Sec. 443 (p. 436). Scale Drawings—Scale drawings have many applications. Everyone should know how to read and make them.

If desired, more exercises on scale drawings can be found in Chapter 18.

Secs. 444-446 (p. 438). Theorems 77, 78, 79—Theorems 77, 78, and 79 can be expressed by the law of cosines. (See Ex. 30, page 502.)

Ex. 4, page 439, can be used to prove that the sum of the squares of the four diagonals of a parallelepiped is equal to the sum of the squares of the twelve edges of the parallelepiped.

Sec. 447 (p. 440). Formulas for the Median and Angle Bisector of a Triangle—These formulas are given without proofs. Some exceptional students may prove them.

The exercises on pages 441-442 offer opportunities for boys and girls who enjoy using their hands:

Some student may wish to make a pantograph; the class may enjoy finding a distance as in Ex. 7; or some student may wish to make a pinhole box camera.

Secs. 448-449 (p. 443). Space Geometry—These sections are included for teachers who want to integrate plane and solid geometry.

Chapter 14. Regular Polygons and the Circle

Objectives:

To extend the logical system of geometric thought to include the relationship of a circle to the regular polygons inscribed in and circumscribed about it.

To teach the more important formulas relating a circle to its polygons.

To provide some enrichment material concerning ways in which nature makes use of the Golden Section.

Time required: 6-10 days.

Planning the content of various courses:

Basic course—§§457-482, 487-491. §§483-486, 492-503 optional. Students should be able to prove the theorems marked with two stars. Theorems and corollaries marked with one star should be completely proved in class. Unmarked theorems and corollaries may be proved informally. Students should memorize and be able to use the formulas in §§467, 482, 488, 490. Construction problems should be studied in class. Exercises should be chosen from A and B groups.

Maximum course for students who plan to specialize in mathematics, science, engineering, and similar fields—§§457-482, 487-496. §§483-486, 497-503 optional. Theorems, corollaries, formulas, and problems should be treated as in the basic course. Exercises should be chosen from the C as well as from the A and B groups.

Minimum course—§§457-471, 479-482, 487-491. Other topics optional. Students should be able to prove Theorem 84. Theorems 87 and 88 may be accepted without proof. Theorems and corollaries, other than Theorem 87, marked with one star should be proved in class, and unmarked theorems discussed informally. Constructions should be discussed informally. Most exercises should be chosen from the A groups.

Specific suggestions for teaching the various topics:

Sec. 457 (p. 451). Need for a Review—Since the work of this chapter requires a background of many previous chapters, a review is important at this time. The exercises on page 451 give a review of some definitions, theorems, and principles which are needed in the work which follows.

Sec. 458 (p. 452). Theorem 80—Call attention to the statement of Theorem 80. Ask the students which statement of the proof depends upon the hypothesis that the angles of ABCDEF are equal; upon the hypothesis that the sides of ABCDEF are equal.

The exercises at the bottom of page 452 are direct applications of Theorem 80.

The pictures on page 453 illustrate the uses of a regular hexagon in architecture and design.

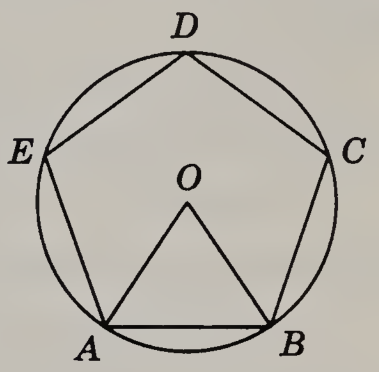
Students are usually interested in bringing to class photographs and newspaper and magazine clippings of geometric designs. Ask them to observe the use of regular polygons in designs of public buildings and churches.

Sec. 459 (p. 454). Theorem 81—Note that the proof of Theorem 81 is based upon Theorem 80. Students should remember the sequence of the two theorems.

Sec. 460 (p. 454). Corollary 1—The proof of this corollary follows from the proof of Theorem 81. The point O is the center of both circles.

Sec. 461 (p. 454). Names Associated with a Regular Polygon—Students have no difficulty in remembering the meaning of the center of a regular polygon but they may forget the meaning of the radius of a regular polygon. They may confuse it with the apothem. Stress the fact that only polygons which are regular have radii.

The drawing at the right may be used to help students understand why a central angle of a regular polygon is so defined.



Sec. 462 (p. 455). Corollary II—This corollary may be proved informally. By drawing the radii of the regular polygon, n congruent triangles are formed. The sum of the n equal angles is 360°.

The A exercises on page 455 are easy. Some students may need help on Exercise 7. Exercises 8, 9, and 10 can be assigned to above average students. Superior students can solve Exercise 11.

Sec. 463 (p. 456). Theorem 82—When proving Theorem 82, students should be careful to prove ABCDE both equilateral and equiangular.

Sec. 464 (p. 456). Corollary I—When proving Corollary I, we can use §286 and Theorem 82.

Sec. 465 (p. 456). Corollary II—Corollary II follows from Theorem 82. The exercise at the bottom of page 456 is an oral exercise.

Sec. 466 (p. 457). Theorem 83—After students understand the proof of Theorem 83, ask them if any part of the proof is based on Theorem 80; on Theorem 81; on Theorem 82.

The exercises at the bottom of page 457 are not designed for weak students. After a student has proved Exercise 1, ask him if he can prove the statement if the word "pentagon" is changed to "hexagon."

The exercises on page 458 offer a variety of selections of assignments. Pupils should be reminded to use the theorems of this chapter if possible, when proving constructions.

Sec. 467 (p. 459). Theorem 84—Students may be interested in making the figure for Theorem 84 from cardboard. The polygon then can be separated into five triangles by cutting along the radii.

Sec. 468 (p. 460). Theorem 85—When proving Theorem 85, call attention to the two conditions necessary for similarity of polygons.

Sec. 469 (p. 460). Corollary—After proving this corollary a short drill on its applications may be beneficial.

Sec. 470 (p. 461). Theorem 86—The proof of Theorem 86 is not difficult. Students usually can learn it without help from the teacher.

Sec. 471 (p. 462). Corollary—The proof of this corollary can be based on § 469 and step 9 in the proof of Theorem 86.

The number of exercises on pages 462 and 463 makes possible various selections for assignments.

For example, a homework assignment may consist of exercises 1-7, 8, 9, 11, 12, 13 for average students and exercises 1-7, 15, 19, 23, 24 for the more able students.

Secs. 472-474 (p. 463). Method of Measuring the Circle, Constants and Variables, Limits—You may wish to discuss these sections with the students.

When discussing variables and constants, you may wish to state the definitions of variable and constant in terms of sets. For example, Allendoerfer and Oakley in Principles of Mathematics (McGraw-Hill) give the following definitions:

"A letter used to represent an arbitrary element of a set, containing more than one element, is called a variable. If a set contains only a single element, the letter used to represent this element is called a constant."

Exercise 7, page 465, should be interesting to many students. Ask some of the students to solve the problem by algebra.

Sec. 479 (p. 467). Theorem 87—Many teachers prefer to postulate this theorem. If the theorem is postulated, it should be so called.

Sec. 480 (p. 467). Corollary I—This corollary is easily proved.

Sec. 481 (p. 468). Corollary II—This section gives another way of stating Corollary I.

Sec. 483 (p. 468). Historical Note on π—This historical note should be of interest to many students.

Not long ago a member of one of our state legislators introduced a bill to make π = 3. Ask your students to comment on this legislator's knowledge of mathematics.

Secs. 484, 485, 486 (p. 468). Finding the Approximate Value of π, Problem 26, Problem 27—These three sections are intended for your better students. Allow students who have these sections assigned to them to prepare the work at home.

Secs. 488-490 (p. 471). Corollaries I, II, III—Regardless of the treatment of Theorem 88, students should be required to prove these corollaries, either formally or informally.

The examples on pages 471 and 472 are intended to show not only the methods of solution but the use of exact and approximate data in a problem.

Use as many exercises on pages 473-475 as are needed to fix ideas.

Sec. 492 (p. 475). Extreme and Mean Ratio—Be sure that students understand the definition of extreme and mean ratio.

Sec. 493 (p. 476). Problem 28—Problem 28 is used to solve Problem 29.

Exercises 1-6 on page 477 deal with extreme and mean ratio. Exercises 7-9 deal with areas involving circles.

Sec. 494 (p. 478). Problem 29—Your better students will enjoy the proof of Problem 29 although it is long.

Secs. 495-496 (p. 479). Corollaries I, II—These corollaries can be proved in class.

Above-average students should be interested in Exercises 4 and 5 on page 480.

Sec. 497 (p. 481). How Nature Makes Use of the Golden Section—This topic gives an opportunity to arouse the interest of some students.

Sec. 498 (p. 482). The Most Beautiful Rectangle—The most beautiful rectangle, whose construction is given in §498, is the same rectangle as used in the diagram on page 475. This fact can be proved as follows:

By construction, $\frac{AB}{AE} = \frac{AE}{EB}$. By inversion, $\frac{AE}{AB} = \frac{EB}{AE}$. By addition, $\frac{AE + AB}{AB} = \frac{AE + EB}{AE}$. By substitution, $\frac{BC + AB}{AB} = \frac{AB}{BC}$. Then $\overline{AB}^2 = BC(BC + AB)$, or $w^2 = h(w + h)$.

You may be interested in reading The Elements of Dynamic Symmetry written by Jay Hambidge and published by Yale University Press. More information on phyllotaxis may be found in an encyclopedia or college textbook on botany.

Secs. 500, 501, 502 (p. 484). Space Geometry—These sections deal with mensuration of the cylinder, cone, and sphere.

The theorems are to be assumed true without proofs.

Sec. 503 (p. 487). Reasoning in a Circle—Pupils have no great difficulty in understanding the error of reasoning in a circle. You may want to supplement these exercises with some of your own.

Chapter 15. Elements of Trigonometry

Objectives:

To teach the meaning of the sine, cosine, and tangent ratios.

To teach the use of four-place tables of sines, cosines, and tangents.

To teach the finding of any part of a right triangle when two sides, or a side and an acute angle are known.

Included in this Manual, though not in the textbook: The teaching of problems in indirect measurement involving use of more than one right triangle, and the teaching of the formula K = ab sin C for the area of a parallelogram and the formula K = 1/2 ab sin C for the area of a triangle.

Time required: 4 days.

Planning the content of various courses:

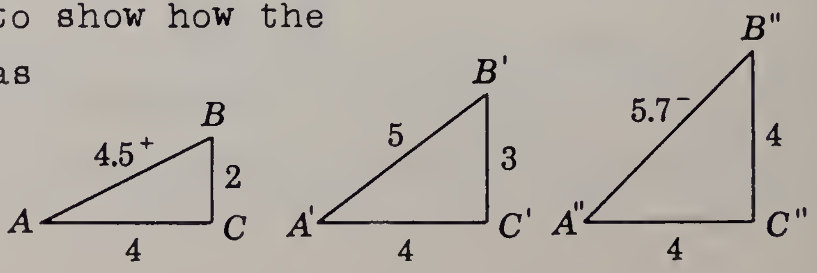
Basic course—§§510-516. Exercises should be chosen from A and B groups.

Maximum course for students who plan to specialize in mathematics, science, engineering, and similar fields—Same as basic course except that more of the B and C exercises should be studied. As optional material you may wish to use the topics on pages 92-94 of this Manual.

Minimum course—Same as basic course except that most exercises should come from A groups.

Specific suggestions for teaching the various topics:

Sec. 511 (p. 493). Trigonometric Functions of an Acute Angle—If the students have not studied numerical trigonometry in a previous course, it may be well to show how the ratios $\frac{BC}{AB}$, $\frac{AC}{AB}$, and $\frac{BC}{AC}$ change as ∠A changes. For example, start with rt. △ACB. Find the values of the three



ratios. Form △A'C'B' by increasing side BC one unit, thus increasing ∠A. Find the values of the three ratios using the sides of △A'C'B' and compare these ratios with those using the sides of △ACB. In like manner form △A"C"B" and obtain ratios.

Let the students discover which ratios increase as ∠A increases and which ratio decreases as ∠A increases.

Call attention to the names of the three other ratios, secant, cosecant, and cotangent.

The exercises are used to fix in mind the meanings of the functions.

Sec. 512 (p. 496). Functions of 45°, 30°, and 60°—Students may be interested in the table at the right. It can be helpful in remembering the values of the functions of the angles listed in the first column.

Angle	sin	cos
0°	$\frac{1}{2}\sqrt{0}$	$\frac{1}{2}\sqrt{4}$
30°	$\frac{1}{2}\sqrt{1}$	$\frac{1}{2}\sqrt{3}$
45°	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{2}$
60°	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{1}$
90°	$\frac{1}{2}\sqrt{4}$	$\frac{1}{2}\sqrt{0}$

Sec. 513 (p. 497). Trigonometric Tables—It is assumed that the students know how to use Table II.

Sec. 514 (p. 497). Interpolation—Some teachers prefer the following form of writing the solution of an interpolation, which will be used to solve Example 2 and Example 3.

Example 2. Find cos 32°10'

Solution.

cos 32° = .8480
60' 10' cos 32°10' = x −.0093
cos 33° = .8387

$\frac{1}{6}$ of .0093 = .0016
x = .8480 − .0016
x = .8464

Example 3. Find tan 45.3°

Solution.

tan 46° = 1.0355
1° .3° tan 45.3° = x .0355
tan 45° = 1.0000

.3 of .0355 = .0107
x = 1.0000 + .0107
x = 1.0107

Sec. 515 (p. 498). Inverse Use of Table—Another form of writing the solutions of Examples 1 and 2 follows.

Example 1. tan B = .2905
Find ∠B.

Solution.

tan 17° = .3057
60' tan B = .2905 .0038 .0190
tan 16° = .2867

$\frac{.0038}{.0190}$ x 60' = 12'
∠B = 16° + 12'
∠B = 16°12'

Example 2. cos B = .3140. Find ∠B.

Solution.

$$\left.\begin{array}{lll} & \cos 71^\circ & = .3256 \\ 60' & \cos B & = .3140 \\ & \cos 72^\circ & = .3090 \end{array}\right. \quad .0050 \quad .0166$$

$$\frac{.0050}{.0166} \times 60' = 18'$$

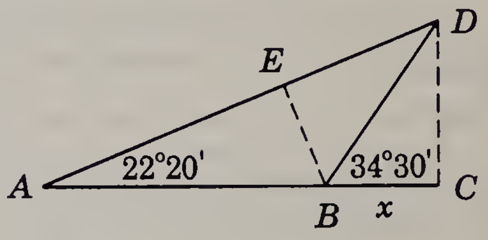
$$\angle B = 72^\circ - 18'$$

$$\angle B = 71^\circ 42'$$

Sec. 516 (p. 499). Finding Parts of a Right Triangle—Students should be reminded that the trigonometric ratios are with a few exceptions, approximate numbers. Therefore the exercises on pages 500-503 have solutions involving computations with approximate numbers.

It may be of interest to the students that the law of sines (Ex. 31) and the law of cosines (Ex. 30) have become more important since the large scale computing machines have been constructed

For the better students supplementary exercises whose solutions involve more than one right triangle may be assigned. The following example and exercises are of this type.



Example. Two points A and B are 180.0 feet apart and in line with the foot c of an antenna CD.

Find the distance BC if ∠ACD = 90°0', ∠DBC = 34°30' and ∠DAB = 22°20'.

Solution 1.

$$\frac{CD}{x} = \tan 34^\circ 30'$$

$$CD = .6873x$$

$$\text{Also } \frac{CD}{x + 180} = \tan 22^\circ 20'$$

$$\frac{CD}{x + 180} = .4108$$

$$\text{Solving, } CD = .4108(x + 180)$$

$$.6873x = .4108(x + 180)$$

$$x = 267.4$$

Solution 2. Draw BE ⊥ AD.

$$\frac{BE}{180} = \sin 22^\circ 20'$$

$$BE = 180 \times .3800 = 68.40$$

$$\angle EBD = 180^\circ - 34^\circ 30' - (90^\circ - 22^\circ 20')$$

$$= 77^\circ 50'$$

$$\frac{BE}{BD} = \cos \angle EBD$$

$$\frac{68.4}{BD} = .2108$$

Solving, $BD = 324.47$

$$\frac{x}{BD} = \cos 34^\circ 30'$$

$$\frac{x}{324.47} = .8241$$

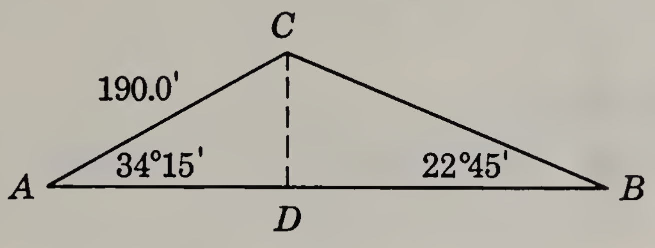
$$x = 267.4$$

Ans. 267.4 ft.

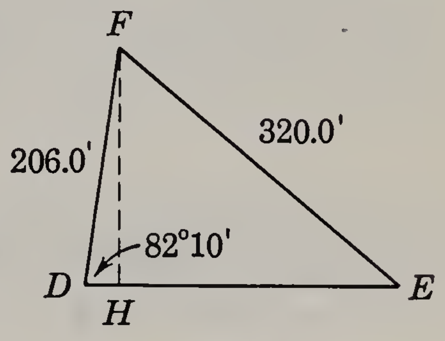
Exercises

1. The top of a cliff is observed from the base and the top of a tower 160.0 feet high. Find the height of the cliff above the base of the tower if the angles of elevation of the top of the cliff from the two observation points are 26°40' and 14°20' respectively.

2. The measurements of a side and two angles of △ABC are shown on the figure below. Find the length of the side BC, using rt. △ADC and rt. △BDC.



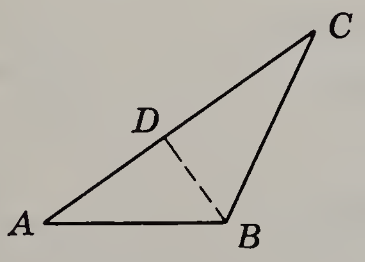
Ex. 2



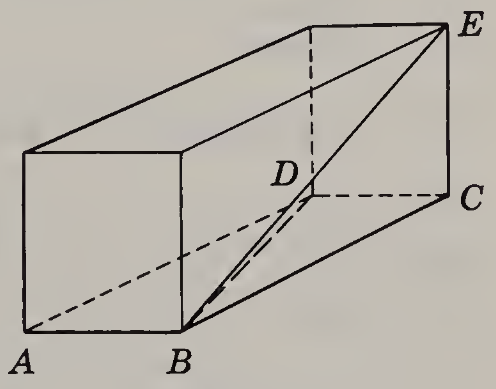
Ex. 3

3. The measurements of side DF, side EF, and ∠D of △ABC are shown on the figure above. Find ∠E.

4. The following are measurements of parts of △ABC: AC = 128.0'; AB = 84.0'; ∠A = 23°30'. Find BC, using rt. △ABD, and rt. △BCD.



Ex. 4

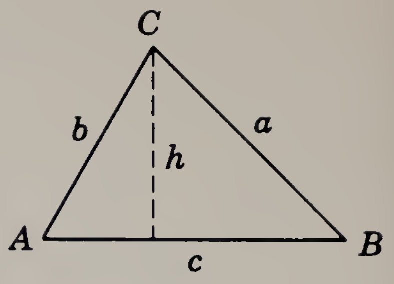


Ex. 5

5. The figure above is a rectangular solid (see page 274). Find CE if by measurement AB = 12.8", ∠CBD = 21°, and ∠CBE = 18°.

For optional work in trigonometry, the area of a triangle and the area of a parallelogram may be studied.

We shall now develop formulas for the areas of these figures.

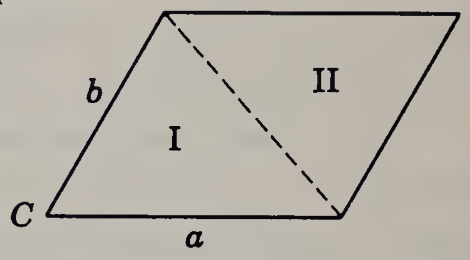


Let ABC be any triangle and let K denote its area.

From C draw the altitude h to the side c. Since $\frac{h}{b} = \sin A$, $h = b \sin A$.

$K = \frac{1}{2}ch$. Then by substituting $b \sin A$ for h, we get $K = \frac{1}{2}cb \sin A$, or $K = \frac{1}{2}bc \sin A$.

Other forms of the formula are $K = \frac{1}{2}ab \sin C$ and $K = \frac{1}{2}ac \sin B$.



Let K denote the area of a parallelogram having adjacent sides a and b and the included angle C.

Draw the diagonal forming △I and △II as shown in the figure.

The area of $\triangle I = \frac{1}{2}ab \sin C$.

Since the parallelogram $= 2\triangle I$, $K = ab \sin C$.

Exercises

In the following exercises the right angle of a right triangle will be denoted by C.

Find the area of each of the following right triangles, given

1. $\angle A = 36°$, $b = 18'$
2. $a = 28.6$, $b = 32.6$
3. $b = 17.4$, $c = 26.2$

Find the area of each of the following triangles given

4. $a = 400$, $b = 500$, $\angle C = 18°40'$
5. $BC = 196$, $AB = 320$, $\angle B = 56°50'$
6. $AC = 540$, $AB = 160$, $\angle A = 26°40'$

Find the area of ▱ ABCD if

7. $AB = 90$, $AD = 70$, $\angle A = 60°$
8. $BC = 124$, $DC = 250$, $\angle A = 44°30'$
9. $AB = 36.1$, $DB = 24.7$, $\angle ABD = 42.3°$

Chapter 16. Inequalities

Objective:

To extend the logical system of geometric thought to include inequalities.

Time required: 5-6 days.

Planning the content of various courses:

Basic course—§§517-534. Theorems 89-91 should be proved in class completely. The other theorems may be discussed informally. Exercises should be chosen from A and B groups.

Maximum course for students who plan to specialize in mathematics, science, engineering, and similar fields—§§517-534. Theorems 89-91 should be proved in class completely, but other theorems may be proved informally. Exercises should emphasize B and C groups as well as A groups.

Minimum course—§§517-523. §§524-534 optional. Exercises should be chosen mostly from A groups.

Specific suggestions for teaching the various topics:

Point out that this chapter begins with a set of five assumptions concerning inequalities, and from these deduces theorems concerning inequalities.

This chapter may be taught as a whole at any time after the students have studied Chapter 10, or it may be integrated as indicated by the footnotes of the text. See pages 121, 155, 330 and 337 of the text.

Sec. 517 (p. 505). Orders of Inequalities—The symbols used in this section are not new to the students. They should be understood before the study of §518.

Sec. 518 (p. 505). Assumptions on Inequalities—At this time it would be well to review Asmts. 9 and 10 on page 54 and Asmts. 42 and 43 on page 288. Attention should be called to the word "positive" in Asmts. 54 and 57.

The exercises on page 506 are intended to clarify and fix the meanings of some of the assumptions on inequality.

Sec. 519 (p. 507). Theorem 89—The proof of Theorem 89 requires no explanation.

Sec. 520 (p. 507). Theorem 90—The key to the proof of Theorem 90 is the drawing of the figure.

The teacher may ask the class to give the contrapositive of Theorem 3, §83 and to compare the contrapositive with Theorem 90.

Sec. 521 (p. 508). Theorem 91—Ask the students to note the similarity of the figures of §521 and §520. The students may wish to prove Theorem 91 indirectly.

Sec. 522 (p. 508). Corollary I—This corollary was stated as an assumption. If time permits, ask the class to show that there is no reasoning in a circle when Corollary I is proved by Theorem 91.

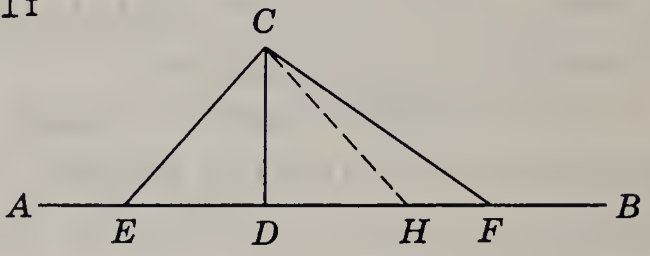
Sec. 523 (p. 508). Corollary II—This corollary can be proved in various ways. It can be proved by using §521, §522, §250, or by the indirect method.

Exercises, pages 508-509. A typical assignment for a class of average ability may consist of exercises 1-5, and 8 for all students and two additional exercises from the C group for the more able students.

Secs. 524, 525 (p. 510). Theorems 92, 93—Note that Theorem 93 is a converse of Theorem 92 and that Theorem 92 is a converse of Theorem 93. Since Theorem 93 has been proved, the corresponding inverse of Theorem 92 is true:

If two triangles have two sides of one equal respectively to two sides of the other and the included angle of the first is not greater than the included angle of the second, the third side of the first is not greater than the third side of the second.

Sec. 526 (p. 512). Theorem 94—If desired, the class may be asked to prove Theorem 94 using the figure shown here.



Suggestions. Make DH = ED; prove CH = CE; prove CF > CH.

Sec. 527 (p. 513). Theorem 95—This theorem can be proved by the indirect method or it can be proved in a way similar to the proof of Theorem 94.

Secs. 528-534 (p. 513). Theorems 96, 97, 98, 99, 100, 101, Corollary—In this text Theorem 96 should not be taught before the chapter on Measurement of Angles and Arcs since the second statement of the proof of Theorem 96 requires §315 as a reason.

Chapter 17. Coordinate Geometry

Objectives:

To introduce students to the study of coordinate geometry.

To emphasize the relationships between algebra and geometry.

Time required: approximately 7 days.

Planning the content of various courses:

Basic course--§§535-549. The formulas of §§539, 540, 544-548 should be derived and discussed in class.

Maximum course--Same as basic course.

Minimum course--Same as basic course.

Specific suggestions for teaching the various topics:

This chapter may be taught after teaching Chapter 13, Chapter 14 Chapter 15, or Chapter 16, or it may be integrated with appropriate subject matter of the preceding chapters. For example, pages 518-523 may be taught while the students are studying parallel and perpendicular lines.

Sec. 535 (p. 519). Cartesian, or Rectangular, Coordinates—In this section the students should learn the definitions of axes, origin, abscissa, ordinate, and coordinates. They should be shown that it is customary for the coordinates to be given in a definite order and know that the ordered pair (2, 3) means $x = 2$ and $y = 3$. They should know that an hypothesis of every analytic geometry example is the coordinate grid, where all lines parallel to the y-axis (x-axis) are perpendicular to the x-axis (y-axis) and are equally spaced.

Assign as many exercises on page 520 as are needed to fix ideas on plotting points.

Sec. 536 (p. 520). Inclination of a Line—You can increase student understanding of the slope of a line by using two yardsticks, one yardstick to represent the x-axis and the other to represent the line.

Sec. 537 (p. 521). The Slope of a Line—When teaching coordinate geometry, it is desirable to have a coordinate grid on the chalkboard. The symbols Δy, Δx, and $\frac{\Delta y}{\Delta x}$ should be understood by all students.

The students should remember that the slope of a line passing through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is the ratio $\frac{\Delta y}{\Delta x}$.

In this work on slopes, the students should always make Δx positive.

Sec. 538 (p. 524). Before beginning the proofs of this section, you may lead the students to prove the following theorems informally.

Theorem. Two points having the same abscissa (ordinate) lie on a line parallel to the y-axis (x-axis).

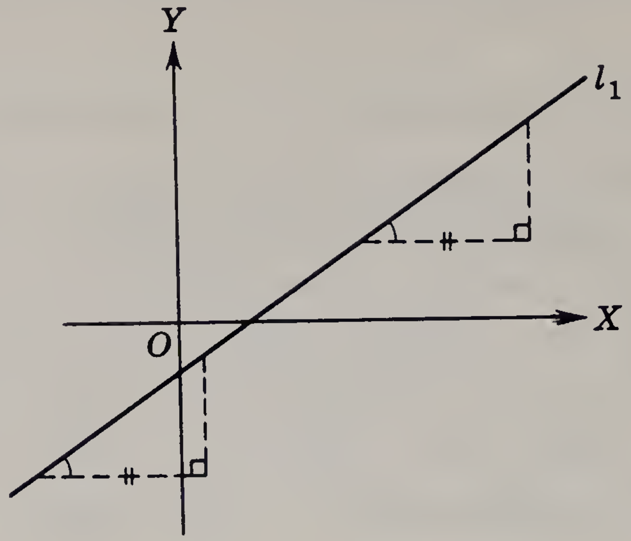
Theorem. The distance between two points having the same abscissa (ordinate) is equal to the difference of their ordinates (abscissas).

We represent this distance by $y_2 - y_1$ or Δy. ($x_2 - x_1$ or Δx).

Theorem. The set of points (x, c) where x is a real number and c is a constant may be represented by $y = c$.

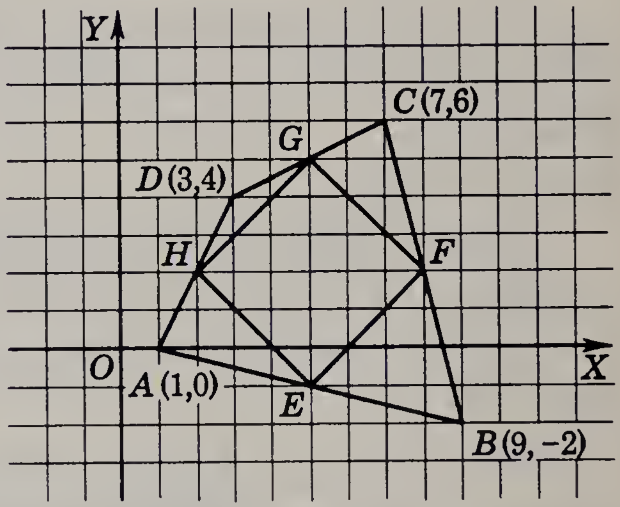
Theorem. The set of points (c, y) where y is a real number and c is a constant may be represented by $x = c$.

Theorem. Any two segments of the same line have the same slope.



Sec. 539 (p. 526). The Midpoint of a Line Segment—The proof of Theorem 102 may be formal, as given in the text, or it may be developed informally. The exercises on page 527 are applications of the theorem. In addition to these three exercises, the following exercise may be assigned.

Exercise. Plot A(1, 0), B(9, -2), C(7, 6), and D(3, 4). Join the points to form polygon ABCD. Find the coordinates of the midpoints of the sides of ABCD. What kind of quadrilateral is formed by joining these midpoints in succession? There are different ways of proving that EFGH is a parallelo gram, one of them being by § 223a. After having studied § 540, we can prove EFGH a square by § 540 and §185.



At this stage we can prove that EFGH is a square by showing that the diagonals EG and HF are equal and perpendicular to each other.

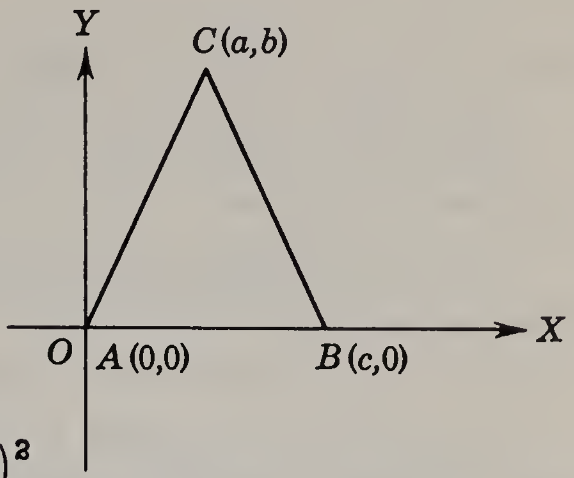
Sec. 540 (p. 527). The Distance between Two Points—The first paragraph of this section gives a review of finding the distances between two points having the same abscissas (ordinates).

The proof of Theorem 103 is easy. The proof can be studied as soon as students have proved the Pythagorean theorem.

The exercises on page 529 offer applications of Theorem 103.

Sec. 541 (p. 529). Proving Theorems by Coordinate Geometry—To this point the exercises have involved figures with vertices of the type (4, 3), (0, 7), and (-3, 2). In this section the figures are more general, having vertices of the type (a, 0), (0, c), and (a, b).

Students should study the examples on pages 529, 530. It is important that they learn how to choose coordinates for the vertices of the given figures.



Some students may be interested in proving Theorem 36 by the methods of coordinate geometry:

Given △ABC, A(0,0), B(c,0) and C(a,b), such that $\overline{AC}^2 + \overline{AB}^2 = \overline{BC}^2$.

To prove that ∠ABC is a rt. △.

Suggestions.

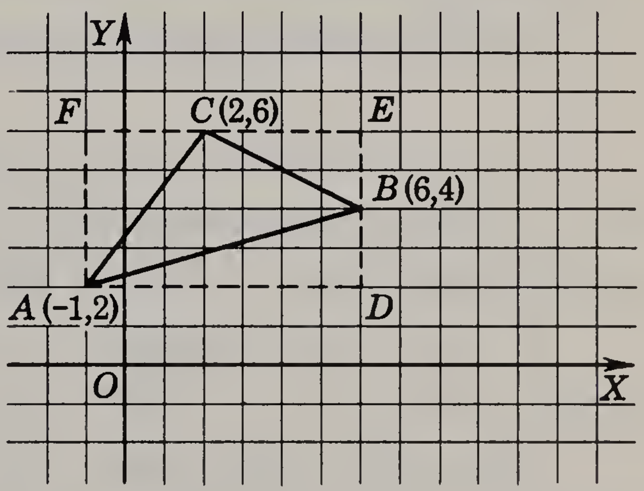
$$\left(\sqrt{(a-0)^2+(b-0)^2}\right)^2 + \left(\sqrt{(c-0)^2+(0-0)^2}\right)^2 = \left(\sqrt{(c-a)^2+(0-b)^2}\right)^2.$$

Simplifying, we get $2ac = 0$. Since $c \neq 0$, $a = 0$. Then c is on OY, and △ABC is a right triangle.

Areas. The application of coordinate geometry in finding areas can be studied after the students have studied § 261 in the text.

The figure of §227, page 237, will help them understand how to find areas of rectangles using the coordinate grid. In the text §260 shows how areas may be found by addition.

Two solutions will now be given for finding the area of a figure where addition and subtraction are used.

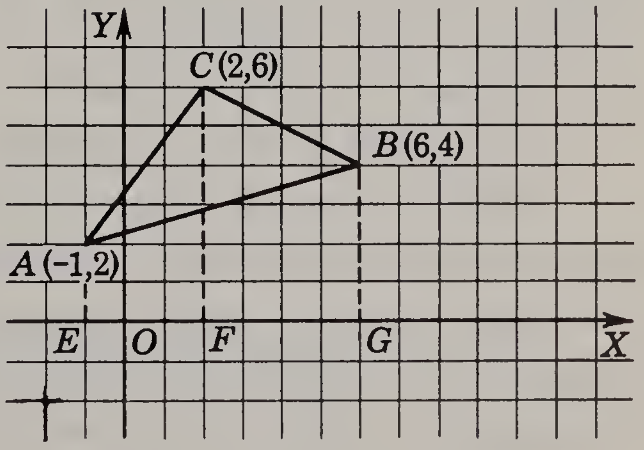


Example. Find the area of △ABC, the vertices being A(-1,2), B(6,4), and C(2,6).

Solution 1. Form rectangle ADEF.

△ABC = ADEF - △ABD - △BEC - △ACF

$= (7 \times 4) - \frac{1}{2}(7 \times 2) - \frac{1}{2}(4 \times 2) - \frac{1}{2}(4 \times 3) = 11.$



Solution 2. Draw ⊥s from A, B, and C to OX.

△ABC = EFCA + FGBC - EGBA

$= \frac{1}{2} \times 3(2 + 6) + \frac{1}{2} \times 4(6 + 4) - \frac{1}{2} \times 7(2 + 4)$

$= 11.$

The following exercises on area may be assigned.

Exercises

1. Plot A(-2,3), B(4,2), and C(4, 7). Find the area of △ABC.

2. Plot A(-1,-2), B(6,-2), C(11,4), and D(4,4). Why is ABCD a parallelogram? Find the area of ABCD.

3. A triangle has one vertex at (0,0) and another at (a,0). How does the area of the triangle vary as the third vertex moves along the line y = c? as it moves along the line x = k?

4. Plot the points A(0,0), B(a,0), and C(e,f). Draw the triangle ABC. How is the area of the triangle affected when

(a) a is doubled and e and f are unchanged?

(b) e is doubled and a and f remain constant?

(c) f is doubled and a and e remain constant?

Sec. 542 (p. 531). The Graph of a Linear Equation—This section gives students drill in graphing equations.

Sec. 543 (p. 532). Linear Loci—This section relates to loci. The use of inverse, converse, and contrapositive theorems can be used as time and occasion permit. The exercises offer practice in fixing ideas.

The following exercises can be used in proving a locus problem.

Exercises

1. Find the equation of the locus of points equidistant from (-3,4) and (5,4). Prove your answer.

2. Find the equation of the locus of points equidistant from (3,1) and (3,-7). Prove your answer.

Secs. 544, 545, 546, 547 (p. 533). The Point-Slope Form, The Slope-Intercept Form, The Two-Point Form, The Intercept Form—The proofs of the formulas in these sections are easy. The exercises are needed to fix ideas.

The teacher may formulate other problems, such as:

Find the area of the quadrilateral formed by the graph of y = x - 2, the line y = 0, the line x = 6, and the line x = 9.

Sec. 548 (p. 537). Equation of a Circle—This section may require more explanation than that required in the four preceding sections.

Some of the more able students may have time to study translations.

Sec. 549 (p. 538). Compound Loci — This section may be integrated with page 356 of the text.

Chapter 18. Additional Topics

Objectives:

Aeronautics—To give the students an introduction to the geometry of flight.

Geometric Relations—To provide a test of a student's power in space perception.

Orthographic Projection—To provide a test of a student's power in space perception, and to give him an introduction to orthographic projection.

Time required:

Since these topics are optional, no time limits are suggested. Some teachers will wish to use the topics as enrichment material to be studied by particular students. Others may want to omit other topics to make room for these. Some will not wish to use them.

Planning the content of various courses:

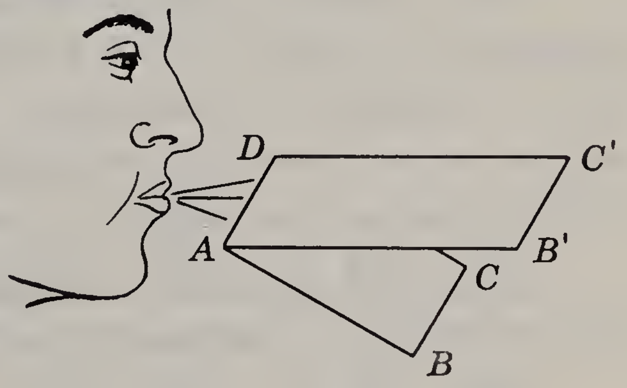
Since the value of these topics will vary from student to student, no attempt has been made to suggest courses.

Specific suggestions for teaching the various topics:

Sec. 550 (p. 541). Why a Kite Flies—Students should know that the directed line segments in the figures at the bottom of page 541 are called vectors and that the parallelograms are called parallelograms of forces.

Sec. 551 (p. 542). Why a Plane Flies—Many students do not know why there is a difference of atmospheric pressure between the space above the wings of an airplane (in flight) and the space below the wings. You can show the difference in pressure easily in class as follows:

Hold a piece of paper ABCD with your fingers along AD and blow air across the top surface. The paper will take the position AB'C'D. A difference in air pressure between the two surfaces of the paper is caused by the velocity of the air on the upper surface.



For another experiment, suspend a ping-pong ball by a thread so that the ball is free to swing. Then with a drinking straw blow air past the ball. The ball will be drawn toward the air current. If a tank of compressed air is available, the ball may be kept in the air by a current of air flowing upward.

These experiments illustrate Bernouilli's principle, which is:

The pressure of a fluid (either gas or liquid) decreases at points where the speed of the fluid increases.

When an airplane has sufficient speed, there is a difference of speed of the air (in reference to the plane) between the upper and lower surfaces of the wings of the plane. This difference of speed causes a difference of pressure and the difference of pressure gives a lifting power to the wings.

Ask the students why the velocity of the air above the wings is greater than the velocity of the air below the wings.

Sec. 552 (p. 544). How Navigators Express Time—Most students are familiar with the navigator's method of counting time. A class discussion of the value of this method may be valuable.

Secs. 553, 554 (p. 544). Meaning of Direction, Direction of the Wind—Students will need to know the meanings of direction of a ray, direction of a point, and direction of the wind. Otherwise they will not understand the problems which appear later in the text.

Sec. 555 (p. 545). Bearing of a Point—If the students have studied §§553 and 554, this section will not be difficult.

Sec. 556 (p. 546). Methods of Navigation—If your class has students interested in flying, you may have one who is willing to discuss some phase of air navigation.

Sec. 557 (p. 547). How to Establish a Fix—Graphic solutions of the problems are suggested for average students. In some cases trigonometry can be used by the more able students.

Sec. 558 (p. 549). How the Wind Affects the Direction and Speed of a Plane—This section needs no explanation. Yet attention may be called to the power of strong winds. You may ask why the walls of a house fly outward in a hurricane; why a boy can ride a bicycle when the wind is perpendicular to his path.

Sec. 559 (p. 549). Speed, Track, and Course—Call attention to the difference between track and course.

Sec. 560 (p. 549). How to Find the Course, Ground Speed, and Drift Angle—A compass course protractor will be useful in measuring angles.

The exercises in this section, if solved graphically, involve the construction of a parallelogram when two sides and the included angle are given.

Sec. 561 (p. 552). How to Find the Ground Speed and the Wind-Correction Angle—The exercises in this section involve the construction of a parallelogram when a diagonal, one side, and the included angle are given.

Sec. 562 (p. 553). Interception—With the help of a student, a teacher can dramatize interception.

Following the pattern shown in the diagram on page 553, the teacher can represent the plane and a student can represent the boat. By this arrangement the teacher can adjust his speed to that of the student and maintain a constant bearing from the student.

Sec. 563 (p. 555). Radius of Action—Note that the drawing on page 556 does not have a parallelogram. The solution of the example makes use of §378c.

Exercise 1, page 556, can be solved by arithmetic. The following problem can be solved by algebra.

How far can a pilot go and return in 3 hours if his air speed is 300 m.p.h., and he has a tail wind of 20 m.p.h. going and a headwind of 30 m.p.h. returning?

ANSWERS

TO NUMERICAL EXERCISES IN TEXT

Pages 8-9. 2. (1) An infinite number. (2) An infinite number. 3. Only one line can be drawn through two given points. 6. (1) Yes. (2) An infinite number. 7. A straight line. 8. AO, AC, OC (any two). 9. n. 10. 6.

Page 10. 1. (1) Two. (2) One. (3) None. 2. AD, AB, BC, AC, AO, OC, BO, OD, BD.

Page 14. 1. (1) 9 in. (2) $9\frac{1}{8}$ in. (3) Yes. 2. $4\frac{4}{8}$ in. 3. (1) AB and EF are approx. equal. (2) CD. (3) GH. 4. AB and DC; AD and BC.

Pages 15-16. 3. AB, AC, BC, BD, CD. 4. BC. 5. BC, CD. 6. CD. 7. (1) OA; OD; AD. (2) OB; OE; BE. 8. EF. 9. AD. 10. OB. 11. EF. 12. OC.

Pages 18-20. 1. (1) No. (2) Yes. (3) Yes. 2. (1) ∠a, ∠b, ∠c, ∠d. (2) ∠TOH, ∠ROT, ∠MOR, ∠HOM. 5. (1) ∠DHE. (2) ∠BKE. (3) ∠FKB. (4) ∠AKF. (5) ∠CHF. 6. (1) 2 lines. (2) 16 angles. 7. (1) 10 angles. 9. ∠ODC. 10. ∠OCB. 11. ∠OBC. 12. ∠OAD. 13. ∠AOD. 14. BO + OD. 15. (1) OB, AD, BC. (2) BC, OB, DO. (3) OC. (4) DC. 16. (1) ∠BOC. Also ∠OAD, ∠OCB. (2) ∠DOC. 19. ∠ABF. 20. ∠DBF. 21. ∠FBD. 22. ∠ABE.

Pages 21-22. 2. Half. 4. a. Right ∠; acute ∠. b. ∠ACD, ∠BCD; ∠ADC, ∠BDC. c. Point D. d. CD. e. ∠BDC. f. CD. 5. Acute; right; straight; obtuse. 6. (1) ∠ABC, ∠BCD, ∠BFE, and ∠CFE. (2) ∠CDE and ∠FEA; ∠EAB and ∠DEF. 7. ∠WST; ∠SWR. 8. No. 10. PR and PS are ⊥ to AB and BC, respectively.

Pages 23-24. 2. $\angle PMN = 66\frac{1}{2}°$; $\angle PNM = 66\frac{1}{2}°$. 4. 180°. 5. 360°. 6. They each equal 64°, approx. 7. 51°15'. 8. 22°35'15". 9. 62°13'57". 10. 18°19'. 11. 14°3'35". 12. 71°27'35".

Page 25. 1. 60°; 50°; 70.8°; 57°42'; (90 - x)°; (108 - 2x)°; (83 - 3x)°. 2. 120°; 135°; 143°; 62°; 149°15'; (180 - x)°; 140° - x°; (135 - 3a)°. 3. 30°, 60°. 4. 72°. 5. $62\frac{2}{3}°$. 6. (1) 40°; 40°; 140°. (2) No. (3) Yes. 7. 45°.

Page 26. 1. Yes; yes. 2. At only one point. 3. Yes. 5. One. 9. 90°. 11. 42° and 48°. 12. 24° and 16°.

Page 27. 1. (1) 2. (2) 2; 2. 2. Diameter. 3. Yes.

4. No. 5. (1) Radius; radius; diameter. (2) An infinite number. 6. (1) Yes. (2) Yes. (3) No. (4) Yes.

Pages 27-28 (Review). 1. None; one; two. 2. (1) $\angle CAB$ or $\angle BAC$. (2) $\angle ACB$ or $\angle BCA$. (3) A line segment. 3. Bisect. 4. $\angle x$. 5. An infinite number; one; none if the points are not in a straight line. 6. (1) AC. (2) OB. (3) $\angle DAB$. (4) Adjacent (5) Vertical. 11. (1) An infinite number. (2) Only one. 12. Complementary; supplementary. 13. Adjacent. 14. Right; acute; obtuse. 15. None; two; none. 16. Bisector. 17. Yes. 20. Midpoint.

Page 29 (Test 1). 1. T. 2. T. 3. T. 4. F. 5. T. 6. F. 7. F. 8. T. 9. F. 10. F. 11. F. 12. T. 13. F. 14. T. 15. T. 16. T. 17. F. 18. T. 19. F. 20. F.

Pages 33-35. 1. (1) 50°; 60°; 100°; 117°; 33°. (2) 360°; 360°. 2. (1) 180°. (2) No. 4. (1) Yes. (2) greater, side. (3) No. 5. The sum of the angles is 360°. 9. (1) AC is longer. (2) BC is longer.

Pages 37-38. 1. Not necessarily. 2. Not valid. 3. Chicken. 4. (1) No. (2) No. (3) Several hundred at least. (4) No.

Pages 40-41 (Review). 1. Through observation. 2. (a) F. (b) F. (c) F. 3. No. 4. $\angle a$: 30°, 37°, 45°. $\angle c$: 83°, 35°, 45°. $\angle a + \angle c$: 113°, 72°, 90°. $\angle r$: 113°, 72°, 90°. 5. Egypt, Babylonia. 6. measurement, observation. 7. Euclid. 8. inductively.

Page 42 (Test 2). 1. determining possible new truths. 2. cannot. 3. cannot. 4. Probably not. 5. $\angle x$: 122°, 90°, 148°. $\angle y$: 122°, 90°, 148°.

Page 43 (Comprehensive Test). 1. line segment. 2. induction. 3. diameter. 4. radius. 5. vertical angles. 6. equal angles. 7. degree. 8. Euclid. 9. ray. 10. supplementary angles. 11. complementary angles. 12. bisect. 13. adjacent angles. 14. perpendicular. 15. straightedge.

Pages 64-65. 1. $x = 8$. 2. $y = 5$. 3. $x = -6$. 4. $x = 2$. 5. $p = -15$. 6. $x = 8$. 7. $y = 9$. 8. $x = -1$. 9. $x = 1$. 10. $x = 6$. 11. $m = 80$. 12. $x = -12$. 13. $y = 12$. 14. $x = -2$.

Page 71. 1. 100°; 80°; 80°. 2. 55°; 125°; 125°. 3. 30°45'; 149°15'. 4. $x°$; $(180 - x)°$. 5. $(x - 10)°$; $(190 - x)°$. 7. $\angle x = 120°$; $\angle 2y = 60°$; $\angle (2y + 60)° = 120°$.

Pages 72-74 (Review). 1. One. 2. One st. $\angle$, or 180°.

3. Hypothesis. 4. Conclusion. 5. Yes. 6. Yes. 7. No. 8. Adjacent and supplementary. 9. No. 10. No. 11. A reason. 12. Yes. 13. Not necessarily. 14. No; no; no; yes. 15. 162°. 16. One. 17. One. 18. 360°, or 2 st. ∠s. 19. They are supplementary. 20. Perpendicular. 21. They lie in a straight line. 24. specific, conclusion. 25. assumptions, definitions.

Page 78. 1. Scalene, 4; isosceles, 1, 2, and 5; equilateral, 2 and 5; right, 3; acute, 1, 2, and 5; oblique, 1, 2, 4, and 5; equiangular, 2 and 5.

Pages 80-81. 1. (1) Legs, AC and BC; hypotenuse, AB. (2) Legs, AD and DC; hypotenuse, AC. (3) Legs, DB and DC; hypotenuse, BC. 2. (1) RS and RT; RS and ST; RT and ST. (2) ∠R and ∠S; ∠S and ∠T; ∠R and ∠T. 3. ∠A and ∠ABD; ∠A and ∠ADB; ∠ABD and ∠ADB. 4. AB and AD; AB and BD; AD and BD. 5. BC. 6. ∠A and ∠C; ∠A and ∠ADC. 7. SK. 8. RW and RT; RW and TW. 9. (1) Isosceles. (2) HK. (3) ∠R. 10. (1) Right. (2) Hypotenuse.

Page 82. 1. (1) Yes. (2) Yes. (3) Yes. (4) Yes. 2. $1 \cong 5$; $4 \cong 6$; $3 \cong 7$. 3. Yes. 4. Yes. 5. Yes. 6. Yes.

Pages 85-86. 1. Yes. 2. No. 3. Yes. 4. No. 5. No. 6. Yes. 7. No. 8. Yes. 9. No. 10. Yes. 11. Yes. 12. Yes. 13. No. 14. No. 15. (1) $\angle x = \angle y$. (2) Yes.

Page 87. 1. (1) Yes. (2) Yes. 2. Yes.

Pages 92-93. 1. No. 2. Yes. 3. Yes. 4. Yes. 5. No. 6. Yes. 7. (1) Yes. (2) Yes. (3) Yes.

Page 98. 1. CO and DO; AD and BC; ∠A and ∠B. 2. EG and FG; GH and GH; ∠E and ∠F; ∠EGH and ∠HGF. 3. AB and AB; AD and AC; DB and BC; ∠C and ∠D. 4. BD and BD; AD and BC; AB and DC; ∠A and ∠C. 5. OP and OP; NO and MO; ∠M and ∠N; ∠NOP and ∠POM. 6. ∠D and ∠B; ∠DCE and ∠ACB; DC and CB.

Pages 107-109. 6. 36°. 7. 39° in each. 8. 66° in each. 9. (1) BC; AC; AB. (2) ∠C; ∠A; ∠B. 10. QN; MN; MR. 19. 56 ft.

Pages 119-120. 1. (1) Left figure: $\angle x = 125°$, $\angle A = 77°$, $\angle C = 48°$. Right figure: $\angle x = 75°$, $\angle A = 48°$, $\angle C = 27°$. (2) $\angle x > \angle A$; $\angle x < \angle C$; $\angle x$ appears equal to $= \angle A + \angle C$.

Page 122. 1. (1) Yes. (2) Yes. (3) $\angle y = 120°$. $\angle z = 155°$. 2. $\angle x + \angle y > 180°$.

Pages 122-123 (Review). 1. Theorem. 2. Postulate. 3. Equilateral. 4. Isosceles. 5. Hypotenuse. 6. Legs. 8. (1) 3. (2) 3. (3) 3. 10. Yes. 12. No. 13. Not necessarily. 14. AC, $\angle A$. 15. No. 16. $\angle x > \angle B$; $\angle x > \angle C$; $\angle x$ is supp. $\angle BAC$.

Pages 124-125 (Test 5). 1. F. 2. T. 3. F. 4. F. 5. T. 6. T. 7. F. 8. T. 9. T. 10. F.

Page 125 (Test 6). 1. 52°. 2. 37°. 3. 10. 4. 120°. 5. 360°. 6. 95°. 7. 72°. 8. 66°. 9. 48°. 10. 10°.

Page 130. 1. No. 2. $\angle AHG$ and $\angle HGD$; $\angle BHG$ and $\angle HGC$.

Page 134. 2. $AB \parallel CD$.

Pages 137-138. 2. $\angle m = \angle x = \angle y = 51^\circ$, $\angle r = \angle n = \angle w = \angle z = 129^\circ$. 3. $\angle n = \angle w = \angle z = 118^\circ$, $\angle s = \angle m = \angle x = \angle y = 62^\circ$. 4. $\angle m = \angle s = \angle x = \angle y = 55^\circ$; $\angle w = \angle z = \angle n = \angle r = 125^\circ$. 13. $\angle A = 84^\circ$, $\angle B = 96^\circ$. 17. 108°, 72°.

Pages 143-144. 2. No. 3. Yes. 4. 56°. 5. 60°. 6. 55°. 7. 56°. 8. 61°, 73°. 9. 40°, 60°, 80°. 10. 45°, 60°, 75°. 11. 98°; 49°. 12. 79°, 70°. 13. 18°, 54°, 108°. 16. $56\frac{2}{3}^\circ$, $30\frac{2}{3}^\circ$; $92\frac{2}{3}^\circ$. 18. No. 20. 120°, 60°, 120°, 60°.

Pages 147-148. 7. 36°, 72°, 72°.

Page 155. 12 in.

Page 156. 5. 135°.

Page 162. 1. CD = 433, BC = 500, DB = 250, AD = 750.

Pages 163-164. 4. (1) 70°. (2) 70°. 6. 45°. 7. (1) $\angle CBD = 60^\circ$, $\angle CDB = 30^\circ$. (2) $BC = \frac{1}{2}BD$. 8. (1) AB = CD. (2) Lowered. 9. 110°. 10. (1) 115°. (2) 65°.

Page 166. 1. Circles, vase, hexagon. 3. O, H, X. (Any two). 4. H, O, S, X. (Any two). 5. equal.

Pages 167-168. 1. (1) An infinite number. (2) Yes. 3. An infinite number.

Pages 168-171. 3. 30°; 40°. 4. 56°. 8. 60°.

Pages 172-173. 1. A point. 2. Straight line. 4. No. 5. An infinite number; one; none. 6. An infinite number; an infinite number; one. 7. An infinite number; one; one. 8. One; two.

Pages 174-175. 1. Yes. 4. Yes. 5. No. 6. Two. 8. Three. 9. Three. 10. a. Yes. b. 170 ft.

Page 176. 2. One. 3. An infinite number. 4. Yes. 5. No. 6. No. 7. (1) Yes. (2) No. 8. One; one. 9. One; one.

Page 177. 1. No. 2. No; no. 3. No; yes. 4. Yes.

Pages 178-179 (Review). 2. One. 3. Two. 4. Right; obtuse; acute. 5. 180°. 6. Converse. 7. Inverse. 10. Complementary. 11. 45°, 45°, 90°. 12. Alternate interior;

corresponding; alternate exterior. 13. 30° - 60° right triangle. 14. 60°. 15. 135°. 16. 20°, 80°, 80°.

Pages 181-182 (Test 11). 1. 70°; 40°. 2. 8 in. 3. 180°. 4. 45°. 5. 40°; 100°; 140°. 6. 5 in. 7. 40°. 8. $(90 - x)^\circ$. 9. 90°. 10. 110°. 11. 30°, 60°.

Pages 182-183 (Test 12). 1. (c). 2. (a). 3. (d). 4. (c). 5. (d). 6. (c). 7. (a). 8. (d). 9. (c). 10. (c). 11. (a).

Page 204. 4. 29°.

Pages 205-206. 2. Opp. ∠ = 65°. Adj. ∠ = 115°. 3. 45°, 135°, 45°, 135°. 4. Yes; yes; yes. 5. No; yes; no. 10. DO = 6 in., AD = DC = 12 in.

Pages 214-215. 1. 13. 2. 26.5. 3. 4, 5, 6.5.

Pages 216-217. 1. 9 in. 2. 14 in. 3. 23 ft. 4 in. 13. 12 in.

Page 218. 1. (1) $\frac{360^\circ}{n}$. (2) 360°. 2. (1) Triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon, decagon. (2) 360° in each case.

Pages 220-222. 1. 360°; 360°; 360°. 2. 120°; 72°; 60°. 3. (1) 2 st. ∠s, 360°; 3 st. ∠s, 540°; 4 st. ∠s, 720°; 5 st. ∠s, 900°; 6 st. ∠s, 1080°; 7 st. ∠s, 1260°; 8 st. ∠s, 1440°. 4. 108°; 120°; $128\frac{4}{7}^\circ$; 135°; 156°. 5. 360°; 360°; 360°. 6. 70°. 7. Decreases; increases. 8. At least one ∠ is measured incorrectly. 9. 24°, 72°. 10. 4 in. 11. 9; 8; 6; 7; 3. 12. 5; 10; 12; 20. 13. 5; 7; 6; 9. 14. 160°. 15. 80°. 16. 16. 17. (1) 60°. (2) 120°. 18. 6. 19. 720°. 22. (1) 4. (2) 6. (3) A hexagon. 28. 130°, 50°, 130°, 50°.

Page 227. 3. 72 ft.

Pages 228-229. 4. F-AB-D and A-CD-G, E-AB-D and A-CD-H; M-AB-F and G-CD-N, M-AB-E and H-DC-N. 5. M-AB-F and A-CD-H, M-AB-E and A-CD-G. 6. Any two of the following pairs: M-AB-F and E-AB-D, A-CD-H and G-CD-N, M-AB-E and F-AB-D, A-CD-G and H-CD-N. 7. (1) Yes. (2) Yes. 8. An infinite number. 9. One. 10. One. 11. Not necessarily. 12. Yes. 13. Yes. 14. Not necessarily. 15. Not necessarily. 16. Yes. 17. Yes. 18. Yes. 19. None. 20. An infinite number. 21. An infinite number. 22. Yes. 23. When the line ∥ the plane or lies in the plane. 24. One. 25. Yes. 26. The ⊥. 27. Yes.

Pages 231-232 (Review). 3. Rhombuses, squares. 4. Rectangles, squares, isosceles trapezoids. 5. Parallelograms. 6. Rhombuses, squares. 10. Neither, the sum = 2 st. ∠s.

11. (1) 360°. (2) 180°. (3) 540°. (4) 720°. 12. (1) 360°. (2) 360°. (3) 360°. 15. Quadrilateral. 17. Trapezoid. 18. 96 in. 19. They are supp. 21. 100°. 22. 90°. 23. 70°. 24. 36°. 25. 160°. 26. 15. 27. 10 in. 28. 8 in. 29. 8 in. 30. 62°, 46°, 72°. 31. 9 in. 32. 120°. 33. EF = 10; ER = 4; SF = 4; RS = 2. 34. DC = 18; ER = 9; SF = 9; RS = 6. 35. DC = 10; AB = 14; SF = 5; RS = 2. 36. 36°, 144°, 36°, 144°. 37. 10 in. 38. 6 in. 39. 35. 40. 2; 9; $\frac{n(n - 3)}{2}$.

Page 234 (Test 14). 1. 8 sides. 2. is not. 3. 720°. 4. supplementary. 5. equilateral triangle. 6. two parallels. 7. 4 right angles. 8. one half.

Page 235 (Test 16). 1. 130°, 50°, 130°. 2. 10 in. 3. 12 in. 4. 120°. 5. 8. 6. 9 in. 7. 2 in. 8. 32. 9. 18. 10. 95.

Page 238. 1. 6750 sq.ft. 2. \$1920. 3. \$2000. 4. \$124.80. 5. 78.8 in. 6. $2x^2 + 7x - 4$. 7. $40\frac{1}{2}$ sq.in. 8. Doubled. 9. Quadrupled.

Page 239. 1. a. $\frac{2}{3}$. b. $\frac{1}{4}$. c. $\frac{1}{2x}$. d. $\frac{x}{2y}$. e. $\frac{1}{2a}$. f. $\frac{a + b}{a - b}$. 2. a. $\frac{1}{3}$. b. $\frac{1}{2}$. c. $\frac{1}{6}$. d. $\frac{1}{2}$. 3. $\frac{7}{5}$; $\frac{5}{7}$. 4. 5x. 5. 30, 75. 6. 36°, 60°, 84°.

Page 240 (top). 1. 1184. 2. $x^2 - 8x + 16$. 3. a. $\frac{2}{3}$. b. $\frac{1}{2}$. c. $\frac{1}{2}$. d. $\frac{2}{1}$. e. $\frac{3}{2}$. f. $\frac{1}{4}$.

Page 240 (foot). 1. Yes. 2. No. 3. No.

Page 242. 1. 216 sq.in. 2. 198.36. 3. 1584 sq.in. 4. They all have the same area. 5. 56.2 ft. 6. $\frac{2}{5}$. 7. 692. 8. 384 sq.in. 9. 84 sq.in. 11. 24.

Page 244. 1. 102. 2. 158.36. 3. 128 ft. 4. 640 sq.ft. 5. 7 ft. 6. $\triangle ABO = \triangle BOC = \triangle AOC = 36$ sq.in. $\triangle ODB = \triangle BOE = \triangle AOF = 18$ sq.in. 9. $\frac{1}{1}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{4}$. 10. 152. 11. $16\frac{7}{8}$ in.

Pages 245-246. 1. 198 sq.in. 2. 323.4. 3. 56 in. 4. 32 in. 6. 21 ft. 7. $6\frac{6}{7}$ in. 8. 297.5 10. $2a^2b$. 11. 25 in. 12. 5. 13. 102. 14. 65 sq.in.

Page 247. 1. $2\sqrt{2}$. 2. 5. 3. $4\sqrt{2}$. 4. $2\sqrt{15}$. 5. $6\sqrt{2}$. 6. 9. 7. $3\sqrt{2}$. 8. $2\sqrt{5}$. 9. 8. 10. $6\sqrt{2}$. 11. $25\sqrt{3}$. 12. 4. 13. $\frac{1}{2}\sqrt{2}$. 14. $\frac{1}{3}\sqrt{3}$. 15. $\frac{1}{3}\sqrt{6}$. 16. $\frac{3}{5}$. 17. $\frac{1}{5}\sqrt{15}$. 18. $\frac{1}{7}\sqrt{7}$. 19. $\frac{1}{a}\sqrt{a}$. 20. $\frac{1}{a^2}\sqrt{a}$.

Page 249. 1. $x = \pm 5$. 2. $x = \pm 5$. 3. $x = \pm 5\sqrt{2}$. 4. $y = \pm 8$. 5. $x = \pm\frac{20}{3}\sqrt{3}$. 6. $x = \pm\frac{1}{6}$. 7. $x = \pm i\sqrt{23}$. 8. $x = \pm 3$. 9. $x = \pm 2\sqrt{3}$.

Page 250. 1. $x = 3, -2$. 2. $x = -6, 5$. 3. $x = \frac{1}{2}, -3$. 4. $x = \frac{1}{3}, -5$. 5. $x = -5, 5$. 6. $y = 6, -4$. 7. $x = 0, 8$. 8. $x = 0, 5$. 9. $x = -1\frac{1}{4}, \frac{1}{3}$. 10. $x = -\frac{1}{3}, \frac{1}{2}$. 11. $y = -3\frac{1}{2}, 5$.

Page 251. 1. $x = 2, -4$. 2. $x = 2, -18$. 3. $x = \frac{1}{2}, -5\frac{1}{2}$. 4. $x = 7, -1$. 5. $x = 41, -1$. 6. $m = 1, -9$. 7. $x = 1.3, -5.3$. 8. $x = -.2, -2.3$. 9. $y = .3, -3.3$. 10. $x = -4, -6$. 11. $x = 6, 5$. 12. $x = 1 \pm \sqrt{41}$. 13. $x = 1, -1\frac{1}{2}$. 14. $y = 1\frac{2}{3}, -2$. 15. $h = 1\frac{1}{2}, -1\frac{1}{3}$. 16. $p = \frac{1 \pm \sqrt{21}}{2}$. 17. $x = -3 \pm 2\sqrt{2}$. 18. $x = \frac{1 \pm \sqrt{5}}{2}$. 19. $x = 10, -9$. 20. $x = -\frac{3}{2}, 4$. 21. $x = 50, -8$. 22. $y = 2.3, -4.3$. 23. $m = 11, -8$. 24. $p = 5.9, -1.9$. 25. $x = 11, -10$. 26. 6 and 9. 27. 13 in.

Pages 253-254. 1. 64. 2. 289. 3. (1) 49. (2) 81. 4. 24 in. 5. 256 sq.in. 6. 7. 7. 225. 8. 192.

Pages 254-256. 1. a. 25. b. 5. c. 29.1. d. 15.6. 2. $10\sqrt{2}$ or 14.14. 3. 7.1 in. 4. 12 ft. 5. 5. 6. 11.55. 7. $\frac{3}{2}\sqrt{6}$. 8. $4\sqrt{6}$ or 9.80. 9. 51 in. 10. 110.85 in. 13. 664.0 sq.ft. 14. $A = a^2 - 2b^2$. 15. $A = \frac{b}{4}\sqrt{4s^2 - b^2}$. 17. Alt., 104; base, 195. 18. 60. 19. 1.8 in. and 3.2 in.

Page 257. 1. $6\sqrt{2}$. 2. 2. 3. $8\sqrt{2}$. 4. $2\sqrt{6}$. 5. $5\sqrt{3}$. 6. $9\sqrt{3}$. 7. $\frac{15}{2}\sqrt{3}$. 8. $7\sqrt{6}$. 9. $16\sqrt{3}$. 10. $36\sqrt{3}$. 11. $\frac{49}{4}\sqrt{3}$. 12. $\frac{5}{4}\sqrt{3}$. 13. 6. 14. 8. 15. 1. 16. 2. 17. $\frac{8\sqrt{3}\sqrt[4]{3}}{3}$. 18. $\frac{4\sqrt{15}\sqrt[4]{3}}{3}$. 19. $\frac{16\sqrt{3}\sqrt[4]{3}}{3}$. 20. $\frac{20\sqrt{3}\sqrt[4]{3}}{3}$. 21. 166.272 sq.in. 22. $10\sqrt{3}$ in., or 17.32 in. 24. $27\sqrt{3}$ sq.in., 46.76 sq.in. 26. 6 ft. 28. $6\sqrt{5}$, or 13.42.

Page 258. 2. Yes.

Page 259. 1. 126. 2. 84. 3. 210. 4. 360. 5. $6\sqrt{6}$. 6. $8\sqrt{11}$. 7. $7500\sqrt{15}$. 8. $\sqrt{(a^2 - c^2)(c^2 - b^2)}$. 9. $\sqrt{mnp(m + n + p)}$. 11. 20; $24\frac{24}{29}$; $28\frac{4}{5}$.

Page 262. 1. 201.25 sq.ft., or 201 sq.ft., approx. 2. 40.49 ft., or 40 ft., approx. 3. Carpeting $4 cheaper.

4. 42.98 rd., approx. 5. 8.24, or 8.2 ft., approx. 6. 127.26 ft., or 130 ft., approx. 7. (1) 67 ft., approx. (2) 64 ft., approx. 8. 94.8 ft., or 95 ft., approx. 9. 49.05 sq.in., or 49.1 sq.in., approx. 10. 17.002 ft., or 20 ft., approx.

Pages 263-264. 1. 75.9, or 80, approx. 2. 330.24 sq.ft., or 330 sq.ft., approx. 3. 253 sq.ft., or 250 sq.ft., approx. 4. 123.49 sq.in. (exact data). 130 sq.in. (approx. data). 5. 31.7 bu. per A., or 32 bu. per A., approx. 6. 1544 (exact data), 1600 (approx. data). 7. 85,000 sq.mi., approx.

Page 265. 1. 7900 sq.ft., approx. 2. 11 sq.in., approx.

Page 266. 1. 72 in. 2. 48 in. and 96 in. 3. 270.625. 4. Doubled. 5. Nine times as large. 6. Halved. 8. $14\frac{2}{17}$ in. 10. $A = \frac{1}{2}dd'$. 11. $A = \frac{s^2}{2}\sqrt{3}$. 12. 27.35 in.

Pages 269-272. 1. a. 291.2. b. $35\frac{7}{12}$. c. 38.9576. d. $4x^2 + 21x - 18$. e. $8x^2y - 12xy^2$. f. $x^4 + x^2 + 1$. 2. $a + b$. 3. a. 27.1. b. $106\frac{2}{3}$ rd. c. $x^2 - 2x + 1$. d. $x + 6$. 4. 2070 sq.ft. 5. 36 ft. 7. 216 sq.ft. 8. 120 sq.in. 9. 48 in. 11. $2\sqrt{30}$ in., or 10.95 in. 12. a. $16\sqrt{3}$ sq.in., or 27.712 sq.in. b. $25\sqrt{3}$ sq.in., or 43.30 sq.in. c. $49\sqrt{3}$ sq.in., or 84.868 sq.in. d. $\frac{27}{4}\sqrt{3}$, or 15.59. e. $\frac{3}{4}\sqrt{3}$, or 1.299. f. $\frac{3}{16}\sqrt{3}$, or .325. 13. 60 rd. by 24 rd. 14. 23.32 ft. 15. $b = 2x - 10$. 16. 406.2 sq.in. 17. 138.56 sq.in. 18. 32 in. 20. 363.4 sq.in. 22. 20 in., 32 in. 23. 12 in., 12 in., $12\sqrt{2}$ in., or 16.97 in. 24. 864 sq.in. 25. 16. 26. $24\sqrt{2}$ in., or 33.94 in. 27. 8 in. 28. $\frac{7}{2}\sqrt{3}$ or 6.06. 29. 39 by 65. 30. 60 ft. x 160 ft. 31. 12.5 in. 32. 14.7 in. 33. 32.3. 34. 96 sq.ft. 35. $A = 4ab$; width = b. 36. 30 ft. 38. 209.92 sq.ft. 39. 180 rd. 40. 14 ft., 18 ft. 41. $96\sqrt{3}$ sq.ft., or 166.27 sq.ft. 42. $A = \frac{b}{4}\sqrt{p^2 - 2pb}$. 43. $A = \frac{h^2}{3}\sqrt{3}$. 44. $4\sqrt{3}$ sq.ft., or 6.928 sq.ft. 45. 194.85. 46. $2\sqrt{5}$ or 4.472, $4\sqrt{5}$ or 8.944, $2\sqrt{15}$ or 7.746. 48. 18 sq.in. 50. 738.7. 51. 540 sq.in. 52. 15 in., 22 in. 53. 256.0 sq.in. 54. 72 sq.in. 55. 34 ft. 56. 45. 58. 68.4. 59. 4.8 ft.

Page 273. 1. (1) $337.50. (2) $228. 2. $612. 3. 30 ft.

Page 274. 2. 6; 5. 3. 12; 12. 4. 8; 10. 9. 28 ft.

Page 275. 1. (1) 216 sq.in. (2) 216 cu.in. 2. (1) 48 sq.in. (2) $16\sqrt{3}$ cu.in., or 27.7 cu.in. 3. A = 336 sq.in. V = 288 cu.in. 4. (1) 384 cu.yd. (2) 160 sq.yd. 5. A = 580 sq.in. V = 800 cu.in. 6. 6 sq.ft. 7. 314,181.8 gal.

Page 276. 3. 6; 12; 16. 4. (1) 15 in. (2) 12.7 in. 5. (1) 4 in. (2) $\sqrt{13}$ in., or 3.6 in. 6. 87.5 sq.in. 7. 113.83 sq.in. 8. $64\sqrt{3}$ sq.in., or 110.85 sq.in. 9. 619.8 sq.in.

Page 277 (top). 1. 4 cu.ft. 2. 3200 cu.ft. 3. 184.75 cu.in. 4. V = 734.3 cu.in. A = 701.28 sq.in. 5. Vol. of prism is 3 times the vol. of pyramid.

Page 277 (foot). 2. 336 sq.in.

Page 279. 1. Overdetermined. 2. Overdetermined. 3. Underdetermined. 4. Determined. 5. Underdetermined. 6. Determined. 7. Overdetermined.

Pages 279-280 (Review). 1. a. $S = bh$. b. $S = bh$. c. $S = bh$, or $S = \frac{1}{2}dd'$. d. $S = s^2$ or $S = \frac{1}{2}d'^2$. e. $S = \frac{1}{2}bh$ or $S = \sqrt{s(s - a)(s - b)(s - c)}$. f. $S = \frac{s^2}{4}\sqrt{3}$. g. $S = \frac{1}{2}h(b + b')$. h. $a^2 + b^2 = c^2$. i. $h = \frac{s}{2}\sqrt{3}$. 2. (1) No. (2) Yes. 3. The ratio of their altitudes. 4. Not necessarily. 5. Right triangle. 6. Not necessarily; not necessarily; yes. 8. Yes. 9. Yes. 10. $\frac{1}{4}$. 11. Quadrupled.

Page 281 (Test 17). 1. F. 2. F. 3. F. 4. T. 5. T. 6. F. 7. T. 8. F. 9. F. 10. T. 11. F. 12. T. 13. F. 14. F. 15. F. 16. T. 17. T.

Pages 282-283 (Test 19). 1. 88 sq.ft. 2. 24 sq.ft. 3. 30 sq.ft. 4. 43.3 sq.ft. 5. 6 ft. 6. 72 sq.ft. 7. 33.3. 8. 15. 9. 72. 10. 12 in., 24 in. 11. 21.3. 12. 80. 13. 10.392. 14. 60. 15. 8.

Page 288. 1. Minor arcs: $\widehat{m}$, $\widehat{p}$, $\widehat{q}$, $\widehat{r}$, $\widehat{KJ}$, $\widehat{KH}$, and $\widehat{HJ}$. The major arc is $\widehat{n}$. 2. $\angle CDE$; $\angle CED$; $\widehat{KJH}$.

Page 289. 2. 60°. 5. Yes.

Page 291. 2. They are equal. 5. 120°, 90°, 60°, 45°. 6. No; yes.

Page 296. 2. Not necessarily.

Page 297. 1. Yes.

Page 299. 2. 15 in.

Pages 301-304. 1. a. $4\sqrt{6}$, or 9.796. b. $2\sqrt{61}$, or 15.62. c. 12. 2. a. 12. b. 16. c. 7. 3. a. 120°. b. 60°. c. 60°. d. 10 in. 4. 14 in. each. 5. 8 in. 6. 248 sq.in. 7. PB = PC = 14". 10. 13.9 in. each. 11. 13 in. 12. 25.5 in. 13. 24 in. 15. .008 in. 17. 6 in. 18. 18. 19. 20 in. 20. 50. 21. 17.49 in., 29.15 in.

Pages 309-310. 1. a. None. b. One common external tangent. c. Two common external tangents. d. One common internal and two common external tangents. e. Two common internal and two common external tangents. 2. None; one; two.

Pages 310-312. 1. 10 in. 6. (1) $EO \perp AD$. (2) $OF \perp BC$. (3) Yes. (4) The radius should be large. (5) Yes. 7. (1) Externally. (2) They are on a straight line.

Page 313 (top). 1. A circle. 2. An ellipse.

Page 313 (center). 1. A circle. 2. An ellipse.

Pages 313-314. 2. $\widehat{AEB}$ is the longer. 3. Yes. 4. Two. 6. An infinite number. 7. One. 8. (1) A great circle. (2) Small circles. (3) The arc of a great circle is shorter.

Page 315. 1. a. T. b. T. 2. a. T. b. F. 3. a. T. b. F. 4. a. T. b. F. 5. a. F. b. F.

Pages 315-316 (Review). 1. Two radii; two chords. 2. (1) An infinite number. (2) One. 3. (1) Two. (2) An infinite number. 4. One; none. 5. 4 inches. 6. A diameter. 7. (1) One. (2) Two. 8. The center of the circle. 9. (1) Radii, diameters. (2) Chords. 10. Tangent; chord or secant. 11. On the circle. 12. Point of tangency, or point of contact. 13. The length of the radius. 14. 12 inches. 15. A secant. 16. Yes; yes; not necessarily. 19. inscribed.

Pages 317-318 (Test 21). 1. tangent to. 2. diameter. 3. the center of. 4. diameter. 5. equal. 6. an external point. 7. common chord. 8. no. 9. one. 10. chord.

Page 319 (Test 23). 1. chord. 2. four. 3. 60°. 4. isosceles. 5. two chords. 6. equidistant from the center.

Page 322. 1. Yes. 2. Yes. 3. Yes; yes; no. 4. 60°. 5. Chords. 6. It increases in the same ratio. 7. Yes. 8. No. 9. $\widehat{BC}$. 10. $\angle BOC$. 11. (1) Yes. (2) Yes. 12. (1) Yes. (2) Yes.

Page 325. 1. $\angle B = 70°$. $\angle C = 50°$. $\angle A = 60°$. 2. $\widehat{BC} = 80°$. $\widehat{AB} = 92°$. $\widehat{AC} = 188°$. 3. $\angle A = 39°$. $\angle B = 84°$. $\angle C = 57°$.

4. ∠A = ∠C = 45°. ∠B = 90°. 6. 66°, 50°, 64°. 7. AC = $9\sqrt{3}$ in., or 15.6 in. BC = 9 in. 8. 480 sq.in.

Pages 326-327. 1. 42°; 56°. 2. 50°; 94°. 3. 86° and 274°. 4. 74°, 150°, 136°. 6. ∠A = ∠B = 72°. ∠C = 36°.

Pages 328-329. 1. 107°; 110°. 2. 75°. 3. 121.5°. 5. 168°, 118°. 6. ∠A = 41°. ∠BCD = 51°. ∠ACB = 44°. 9. 45°, 90°.

Pages 331-332. 1. 29°. 2. 104°. 3. ∠A = 30°. ∠B = 100°. ∠C = 110°. ∠D = 120°. 4. a. 40°. b. 55°. c. 150°. d. 85°. 5. 270° and 90°. 11. 101° and 51°.

Page 334. 3. Four of the angles are each 41°; the other four are each 49°.

Pages 340-341. 1. Valid. 2. Not valid. 3. Not valid. 4. Valid. 5. Not valid. 6. Not valid. 7. Not valid. 8. Not valid. 9. Not valid. 10. Not valid. 11. Not valid. 12. Not valid. 13. Not valid. 14. Not valid. 15. Not valid. 16. Valid. 17. Not valid.

Pages 341-342 (Review). 2. $\overset{\frown}{AB}$. 3. 90°. 4. Point of tangency, or point of contact. 5. c, a, d, b. 6. No. 7. Yes. 9. Two. 10. Minor. 11. An infinite number. 12. 70°, 35°, 65°, 80°. 13. 21 in. 14. 60°, 120°, 60°, 60°, 30°. 15. 8 in., 16 in., 8 in., $8\sqrt{3}$ in., $8\sqrt{3}$ in. 16. The opposite ∡ are supp. 17. No. 18. 5 in. 19. 39 in. 20. 80°.

Pages 343-344 (Test 25). 1. a right angle. 2. its intercepted arc. 3. is doubled. 4. one half its intercepted arc. 5. acute. 6. supplementary. 7. equal. 8. one half the difference of its intercepted arcs. 9. one half the sum. 10. chords.

Pages 344-345 (Test 27). 1. 86°. 2. 48°. 3. 19°. 4. 94°. 5. 90°. 6. 24°. 7. 72°. 8. 7 in. 9. 45°; 135°. 10. 120°. 11. 88°. 12. 5 in. 13. 100°.

Page 362. OB = OC = 5.

Page 368. 2. $\overset{\frown}{AHC}$ is obtuse.

Page 376. 1. Friday. 3. Smith.

Page 377 (Review). 7. equilateral. 8. center. 13. One. 14. Two. 16. Segment of a circle. 17. 1 to 2. 21. $\frac{4}{3}\sqrt{3}$ in.

Page 379 (Test 28). 1. T. 2. T. 3. F. 4. F. 5. T. 6. T. 7. T. 8. T. 9. F. 10. T.

Page 382. 1. $\frac{2}{3}$. 2. $\frac{2}{5}$. 3. $\frac{m}{2}$. 4. $\frac{1}{4x}$. 5. $\frac{5ac}{6b}$. 6. $\frac{x-4}{4x}$. 7. Yes. 8. Yes. 9. No. 10. No. 11. Yes.

12. Yes. 13. Yes; yes. 14. $x = 16$. 15. $x = 3$. 16. $x = 2\frac{3}{5}$. 17. $x = 2\frac{1}{3}$. 18. $x = 5$. 19. $x = -4$. 20. $x = \frac{ab}{c}$. 21. $x = \pm 4$. 22. $x = \frac{1}{2}$. 23. 3, 4, 6, 8; $a + b$, $c + d$, m, n; $a^2 + 1$, a, $a - 1$, 5. 24. (1) Means, 6 and 15; extremes, 5 and 18. (2) Means, 5 and 7; extremes, 4x and 8. (3) Means, b and c; extremes, a^2 and d. 25. $\frac{4}{x} = \frac{x}{9}$. 26. $\frac{2}{8} = \frac{8}{32}$. 27. a. $x = 28$. b. $x = 2$. c. $x = -1\frac{3}{8}$. d. $x = \pm 5$.

Page 385. 2. Extremes, m and t; means, n and s. 4. a. $x = 5\frac{3}{5}$. b. $x = 1\frac{5}{16}$. c. $x = 7$. d. $x = \pm 8$. e. $x = 3, -2$. f. $x = b$. 6. ±12; ±21. 7. a. 6. b. 15. c. $\frac{1}{6}$. d. $\frac{np}{m}$. 8. a. $\frac{5}{2} = \frac{35}{14}$. b. $\frac{3}{8} = \frac{9}{24}$. c. $\frac{9}{14} = \frac{22.5}{35}$. d. $\frac{n}{m} = \frac{t}{s}$. e. $\frac{b}{a + b} = \frac{d}{c + d}$. f. $3 : x = 5 : y$. 9. a. (a) $\frac{2}{14} = \frac{5}{35}$. (b) $\frac{8}{24} = \frac{3}{9}$. (c) $\frac{14}{35} = \frac{9}{22.5}$. (d) $\frac{m}{s} = \frac{n}{t}$. (e) $\frac{a + b}{c + d} = \frac{b}{d}$. (f) $x : y = 3 : 5$. b. (a) $\frac{7}{5} = \frac{49}{35}$. (b) $\frac{11}{3} = \frac{33}{9}$. (c) $\frac{23}{9} = \frac{57.5}{22.5}$. (d) $\frac{m + n}{n} = \frac{s + t}{t}$. (e) $\frac{a + 2b}{b} = \frac{c + 2d}{d}$. (f) $(x + 3) : 3 = (y + 5) : 5$. c. (a) $\frac{-3}{5} = \frac{-21}{35}$. (b) $\frac{5}{3} = \frac{15}{9}$. (c) $\frac{5}{9} = \frac{12.5}{22.5}$. (d) $\frac{m - n}{n} = \frac{s - t}{t}$. (e) $\frac{a}{b} = \frac{c}{d}$. (f) $(x - 3) : 3 = (y - 5) : 5$.

Page 386. 1. a. $\frac{3}{2}$. b. $\frac{7}{5}$. c. $\frac{1}{4}$. d. $\frac{1}{5}$. e. $\frac{35}{24}$. f. $\frac{b}{a}$. g. $\frac{c + d}{a + b}$. h. $\frac{s - n}{m - r}$. 2. a. $\frac{4}{3}$. b. $\frac{5}{m}$. c. $\frac{4}{1}$. d. $\frac{1}{7}$.

Page 387. 1. Possible answers are: a. 1 ft., 2 ft., 6 in. b. 2 cm., 1 cm., $\frac{1}{2}$ cm. c. $\frac{1}{6}$ ft., $\frac{1}{12}$ ft., $\frac{1}{24}$ ft. d. 0.1 in., 0.05 in., 0.02 in. 2. a. 0.1 in. b. 0.01 in. c. 0.001 in. d. 0.0001 in. 3. No. 4. No. 5. Yes. 6. (1) $\frac{CD}{CE} = \frac{DA}{EB}$; $\frac{DA}{CD} = \frac{EB}{CE}$. (2) $\frac{CD + CA}{DA} = \frac{CE + EB}{EB}$; $\frac{CA}{DA} = \frac{CB}{EB}$. 7. (1) $\frac{4}{3}$. (2) Yes. (4) Yes. (6) $\frac{4}{3}$. (7) Yes. (9) Not necessarily. 8. (1) $\frac{4}{7}$. (2) $\frac{4}{7}$. (3) $\frac{3}{7}$. (4) $\frac{3}{7}$.

Page 389. 1. CE = 2EB. 2. $CE = \frac{2}{3}CB$. 3. 4 in. 4. $12\frac{1}{2}$ ft. 5. CD = 9, DA = 6. 6. $19\frac{1}{5}$. 7. $10\frac{2}{3}$ in. 9. 3 in., 12 in. 11. $3\frac{3}{7}$ in., $2\frac{4}{7}$ in.; $5\frac{1}{7}$ in., $3\frac{6}{7}$ in.

Page 390. 1. $7\frac{1}{2}$ in.

Page 393. 1. (1) $\frac{2}{5}$. (2) $\frac{3}{5}$. (3) 3x. 3. 6 in., 10 in. 4. (1) $\frac{2}{3}$. (2) $\frac{5}{3}$. (3) 5x. (4) 3x. 5. 49 in., 63 in.

Page 394. 1. 4.8. 2. 8.

Page 396. 1. Isosceles. 2. 12. 3. 6 in., 8 in. 4. (1) 9 in., 6 in. (2) $3\frac{5}{9}$ in., $4\frac{4}{9}$ in. (3) $7\frac{19}{23}$ in., $4\frac{4}{23}$ in. 5. (1) Through B. (2) BP = 28, AP = 40. 6. 16, 24. 9. 24 in., 36 in.

Pages 398-399 (Review). 2. No. 5. a, b, and d. 7. a. $\frac{x}{n} = \frac{y}{n}$. b. $\frac{y}{x} = \frac{n}{m}$. c. $\frac{x + y}{y} = \frac{m + n}{n}$. d. $\frac{x - y}{y} = \frac{m - n}{n}$. 8. Any 2 of the following: $\frac{a}{c} = \frac{d}{b}$; $\frac{a}{d} = \frac{c}{b}$; $\frac{c}{a} = \frac{b}{d}$; $\frac{d}{a} = \frac{b}{c}$. 12. ±6. 13. 27. 14. 14.4. 15. 16 and 24. 16. $6\frac{2}{3}$. 17. $10\frac{2}{3}$. 18. $25\frac{1}{5}$. 19. 12 in. and 15 in.

Page 400 (Test 30). 1. F. 2. F. 3. F. 4. T. 5. F. 6. T. 7. T. 8. T. 9. F. 10. T. 11. F. 12. T. 13. T. 14. F. 15. T. 16. T. 17. T. 18. T. 19. F. 20. T.

Page 401 (Test 31). 1. 6. 2. x = 3. 3. $\frac{6}{m} = \frac{5}{2n}$. 4. 3 in. 5. 12. 6. 15. 7. 6. 8. 36°, 54°. 9. 9.6. 10. 10.8. 11. 8. 12. $10\frac{2}{3}$. 13. 8 : c. 14. $b^2 - d^2$.

Page 404. 1. 21 in. 2. 9 ft., 4.5 ft. 3. A'B' = 7.5, B'C' = 3.75. C'D' = 4.5. 4. (1) 6, 7.5, 9, 12. (2) $\frac{2}{3}$. 5. (1) Yes. (2) Yes. (3) Yes. 6. (1) Yes. (2) Not necessarily. (3) 1. 7. Yes. 8. No; no; yes. 11. 8.484 in.

Pages 408-409. 1. (1) Yes. (2) Yes. (3) Yes. (4) Yes. (5) $\frac{1}{3}$. (6) $\frac{1}{3}$. 3. 160 ft.

Page 410. 1. 12. 3. Yes. 4. No.

Page 412. 2. Yes.

Page 415. 2. a. 20. b. 20. c. 14. d. 16. e. $10\sqrt{6}$. 3. a. 6. b. 149.3. c. 32. d. 19.0. e. 28.0. 4. a. 9. b. 8. c. 5. d. 14.2. 5. a. 6. b. 12. c. 18. 8. 12 in., $6\sqrt{6}$ in.

Page 416. 4. (1) 2.4. (2) Geometric.

Page 419. 1. $5\frac{1}{3}$ in. 2. 18 in., 5 in. 3. 20 in. 4. 4 in., 21 in. 5. $22\frac{1}{2}$ in. 6. $9\frac{1}{3}$. 7. 24 in. 8. 12.5 ft. 9. 200. 10. 2 ft. 11. 29.39 in.

Pages 420-421. 1. 4.9 in.; 5.1 in. 2. 12 in. 3. 30.59 in. 4. 36 in. 5. 55 in. 6. 24 in., 26 in.

Page 422. 1. Yes. 2. 52. 3. $5\frac{2}{5}$ in., $7\frac{1}{5}$ in., 9 in., $10\frac{4}{5}$ in., $12\frac{3}{5}$ in.

Page 425. 1. 63 in. 2. 12 in., $13\frac{1}{3}$ in., 16 in., 20 in. 4. (1) 2.5 in. (2) 5.4 in. 5. 50 in. 6. $\frac{a + 3}{3a + 2}$. 7. 6.93 in. 8. 23.09 in. 9. (1) $AC = \sqrt{2}$. (2) $AD = \sqrt{3}$. 10. 10. 13. 12 in.

Page 427. 1. $\frac{16}{25}$. 2. $\frac{4}{5}$.

Page 428. The area of A'B'C'D'E' = 9 times the area of ABCDE.

Page 429. 1. 1 : 2; 1 : 2; 1 : 4. 2. $\frac{1}{9}$; $\frac{1}{3}$. 3. $\frac{4}{1}$. 5. 7.5 in. 6. 48 in. 7. 324 and 729. 8. $\frac{3}{4}$. 9. Yes. 10. $\frac{1}{4}$.

Pages 432-436. 1. 5 in. 2. 30.51 in. 4. 7. 5. 16.58 in. 6. $\frac{9}{1}$. 7. 24 in. 8. 108 sq.in. 9. No. 10. 12 in.; 21.63 in.; 14.42 in. 11. $\frac{4}{5}$. 12. $\frac{5}{6}$; $\frac{25}{36}$. 14. 13. 15. $\frac{9}{1}$; $\frac{1}{1}$. 21. 12.81 ft. 24. $20\sqrt{2}$ in., or 28.28 in. 25. 576 sq.in. 26. 16 in. and 20 in. 27. $\ell\sqrt{d^2 - \ell^2}$. 29. $\frac{c}{4}\sqrt{4b^2 - c^2}$. 38. 20 in., 28 in.

Page 437. 1. Living room, 24 ft. by 22 ft. Dining room, $14\frac{1}{2}$ ft. by $11\frac{1}{2}$ ft. Kitchen, $14\frac{1}{2}$ ft. by 11 ft. Bedrooms: 14 ft. by 13 ft., 13 ft. by 11 ft., and 13 ft. by 10 ft. 3. 50 lb. 4. The resultant of 60° is the greatest.

Page 438. 1. $a = 15$. 2. $c = 91.7$. 3. $BC = 5$. 4. $2\frac{2}{17}$. 5. $6\frac{1}{30}$.

Page 439. 1. Acute. 2. Right. 3. 18.

Page 440. 1. 21.6. 2. 8. 3. (1) $13\frac{7}{11}$ and $16\frac{4}{11}$. (2) 6.8. 4. 8.5, 11.0, 15.5. 5. 15.5, 17.2, 13.7. 6. 9.9, 7.9, 6.6. 7. (1) 12, 9.6, 9.6. (2) 12, 9.2, 9.2. (3) 12, 9.4, 9.4.

Pages 441-442. 1. 193.75 ft. 2. $DB = \frac{2}{3}AC$. 5. $6\frac{1}{2}$ ft. 8. $29\frac{3}{4}$ ft. 9. 80 in.

Page 443. 1. $22\frac{1}{2}$. 2. 16. 3. 3. 4. 15. 5. AB = 8, CD = 12.

Page 444. 1. 48 sq.in. 2. $\frac{1}{9}$.

Page 445. 5. 3. 11. Yes. 13. Equilateral $\triangle$s . 14. Squares. 15. 20. 16. (1) No. (2) Yes. 19. $\sqrt{2} : 1$. 22. Yes.

Page 447. 1. T. 2. T. 3. F. 4. F. 5. T. 6. T. 7. F. 8. F. 9. F.

Page 448. 1. proportion, equal. 2. triangles, similar. 3. corresponding sides. 4. sides proportional. 5. segments, diameter. 6. corresponding sides, proportional. 7. corresponding angles, equal. 8. projection. 9. are similar. 10. secant, external segment.

Page 449. 1. 30. 2. 16. 3. 75 in. 4. $10\frac{1}{2}$. 5. 2. 6. (1) $\triangle$FHG and $\triangle$KLM. (2) $5\sqrt{2}$. (3) 3. 7. 9. 8. 15. 9. 8. 10. 12 in. 11. AD = 9. AB = 9. 12. $9\frac{3}{14}$. 13. 5 in. 14. $\frac{5}{4}$.

Page 451. 2. (1) Equilateral $\triangle$. (2) Square. (3) Regular pentagon. (4) Regular hexagon. 11. polygon, polygon. 12. Yes; yes. 13. No; no. 14. Yes; no.

Page 455. 1. a. 60°. b. 108°. c. 90°. d. 120°. e. 135°. f. $128\frac{4}{7}^{\circ}$. 6. Yes; yes; no. 8. $24\sqrt{3}$ sq.in. 11. $4\sqrt{5\sqrt[4]{3}}$ sq.in.

Page 458. 13. 24 in.

Page 459. 2. $\sqrt{3}$ in.

Page 460. 128 sq.in.

Pages 462-463. 1. 5 in. 2. Perimeter ratio, $\frac{1}{3}$; area ratio, $\frac{1}{9}$. 3. Ratio of radii, apothems, and perimeters = $\frac{3}{5}$. Ratio of areas = $\frac{9}{25}$. 4. $10\sqrt{2}$, or 14.14 in. 5. $384\sqrt{3}$, or 665.09 sq.in. 6. 10.5 in. 7. $\frac{2}{1}$; $\frac{2}{1}$. 8. 0.71. 9. $\frac{3}{8}$; $\frac{9}{64}$. 10. 63 sq.in. 11. 162 sq.in. 12. 3.46 ft. 13. 9.24 ft. 14. $\frac{3}{2}$. 15. $\frac{3}{1}$. 16. $\frac{2}{1}$. 17. $\frac{\sqrt{2}}{1}$. 18. $6\sqrt{2}$, or 8.48 in. 19. 128. 20. Inscribed square: perimeter, $32\sqrt{2}$, or 45.25 in.; area, 128 sq.in. Circumscribed square: perimeter, 64 in.; area, 256 sq.in. 21. No. 23. $162\sqrt{3}$, or 280.58 sq.in. 24. $96\sqrt{3}$ or 166.27 sq.in.

Pages 464-465. 2. (1) The radius of the circle. (2) The circumscribed circle. 3. $\frac{2}{3}$. 4. $\frac{7}{9}$. 5. $\frac{8}{9}$. 6. a. The circle. b. The area of the circle. c. No. 7. No.

Page 466 (top). 2. $m = n$. 3. pr → pc. 4. 180°; 0°.

Page 466 (foot). 1. (1) The circumscribed square. (2) The circumscribed square. 2. The polygon with 16 sides.

Pages 473-475. 1. a. 25.133. b. 37.699. c. 75.398. d. 785.40. 2. a. 28.274. b. 706.86. c. 5541.8. d. 1984.4. 3. a. 4.0000. b. 25.000. c. 11.000. d. 31.831. 4. a. 2.04. b. 2.3936. c. 18. 5. a. 4.7746. b. 7.1620. c. 10.823. 6. 56.549. 7. $\frac{1}{2}$; $\frac{1}{4}$. 8. 2.2568. 9. 87.965 sq.in. 10. 107.29 in., or 107.3 in. 11. 65.133 in. 12. 324 sq.in. 13. $63.62. 14. 194.87 in. 15. 28.540 sq.in. 17. $d = 4$. 18. 22.111 sq.ft. 19. 16 times. 21. 336.13. 24. 737.76. 26. 20.30 sq.in.

Page 477. 2. 3.09 in., 1.91 in. 5. 111.25°, 68.75°. 6. 3.7 in., 2.3 in. 7. 11.18 sq.in. 8. 3.26 sq.in. 9. 36.53 sq.in.

Page 485. 2. a. 150.80 sq.in. b. 125.66 sq.ft. c. 87,965 sq.ft. d. 448 sq.in. 3. a. 942.48 cu.in. b. 7602.7 sq.ft. c. 1256.6 sq.in. d. 36,669 cu.in. or 21.220 cu.ft. 4. $A =$ 785.40 sq.ft. $V =$ 1570.8 cu.ft. 5. 420 sq.in. 6. 4.60 in.

Pages 485-486. 2. $\ell = 13$ in. $S =$ 204.20 sq.in. 3. $V =$ 301.59 cu.in. $S =$ 188.50 sq.in. 4. $S =$ 5127.1 sq.in., or 35.605 sq.ft. $V =$ 13,662 cu.in., or 7.9063 cu.ft. 5. 703.72 sq.in.

Page 486. 1. a. $S =$ 452.39 sq.in. $V =$ 904.78 cu.in. b. $S =$ 201.06 sq.in. $V =$ 268.08 cu.in. c. $S =$ 113.10 sq.ft. $V =$ 113.10 cu.ft. d. $S =$ 314.16 sq.ft. $V =$ 523.60 cu.ft.
2. (1) 452.39 sq.in. (2) 452.39 sq.in. (3) 904.78 cu.in. (4) 1357.2 cu.in. 3. $r = \frac{1}{2\pi}\sqrt{\pi S}$. 4. (1) $\frac{2(r + h)}{r + \ell}$. (2) $\frac{3}{1}$.
5. $2945.25. 6. (1) 615.75 sq.in. (2) 615.75 sq.in. (3) 1436.8 cu.in. (4) 2155.1 cu.in.

Page 488. 1. Incenter, circumcenter, orthocenter, and centroid. 2. a. Yes. b. Not in general. c. Not in general. d. Yes. e. Yes. f. Yes. 3. It must be equilateral and equiangular. 4. Triangle, rhombus. 5. When they have the same number of sides. 6. a. The radius. b. Zero. c. The circumference. d. The area of the circle. e. 180°. f. 0°. 7. 11 by methods given in the text. 8. It is the radius of the circumscribed ⊙. 9. Regular hexagon. 10. a. No. b. Yes. 12. $\frac{2}{5}$, $\frac{4}{25}$. 13. No. 14. d. 15. a.

Pages 490-491. 1. F. 2. T. 3. T. 4. T. 5. F. 6. F. 7. T. 8. F. 9. T. 10. F. 11. T. 12. T. 13. F. 14. F. 15. T. 16. F. 17. F. 18. F.

Page 491. 1. $\frac{16}{25}$. 2. 3.9270 sq.in. 3. $12\sqrt{3}$ or 20.784. 4. 4. 5. 40 sq.ft. 6. $3\sqrt{3}$ or 5.196 in. 7. 90°. 8. 50.266 in. 9. 93.75. 10. $\frac{5}{2}$. 11. 28.540 sq.in. 12. 13.090 in.

Pages 494-496. 1. In △DEF: sin D = $\frac{d}{e}$; cos D = $\frac{f}{e}$, tan D = $\frac{d}{f}$, sin F = $\frac{f}{e}$, cos F = $\frac{d}{e}$, tan F = $\frac{f}{d}$. In △GHK: sin G = $\frac{g}{k}$, cos G = $\frac{h}{k}$, tan G = $\frac{g}{h}$, sin H = $\frac{h}{k}$, cos H = $\frac{g}{k}$, tan H = $\frac{h}{g}$. In △RST: sin T = $\frac{t}{r}$, cos T = $\frac{s}{r}$, tan T = $\frac{t}{s}$, sin S = $\frac{s}{r}$, cos S = $\frac{t}{r}$, tan S = $\frac{s}{t}$. 2. △ABC: sin A = .60, cos A = .80, tan A = .75, sin B = .80, cos B = .60, tan B = 1.33. In △MNP: sin M = .88, cos M = .47, tan M = 1.88, sin P = .47, cos P = .88, tan P = .53. In △XYZ: sin Z = .92, cos Z = .38, tan Z = 2.40, sin X = .38, cos X = .92, tan X = .42. 3. (1) sin T = $\frac{RS}{TR}$, cos T = $\frac{ST}{TR}$, tan T = $\frac{RS}{ST}$. (2) sin T = $\frac{SW}{ST}$, cos T = $\frac{TW}{ST}$, tan T = $\frac{SW}{TW}$. 4. (1) sin R = $\frac{ST}{TR}$, cos R = $\frac{RS}{TR}$, tan R = $\frac{ST}{RS}$. (2) sin R = $\frac{SW}{RS}$, cos R = $\frac{RW}{RS}$, tan R = $\frac{SW}{RW}$. 5. (1) sin 20° = .34, cos 20° = .94, tan 20° = .36. (2) sin 70° = .94, cos 70° = .34, tan 70° = 2.75.

Page 497. 1. .3420. 2. .9703. 3. .9703. 4. .3420. 5. .3057. 6. 1.9626. 7. .2079. 8. .5878. 9. 11.4301. 10. The sine and tangent increase; the cosine decreases. 11. 13°. 12. 26°. 13. 15°. 14. 51°. 15. 82°. 16. 82°.

Page 498. 1. .5925. 2. .7361. 3. .3411. 4. 3.2129. 5. 1.9053. 6. .9784. 7. .1621. 8. .8358. 9. 1.0000. 10. .6639. 11. .6006. 12. ∞ . 13. 1.7405. 14. .2630. 15. 0. 16. .4726.

Page 499. 1. 6°30', or 6.5°. 2. 17°15', or 17.3°. 3. 62°12', or 62.2°. 4. 27°48', or 27.8°. 5. 11°48', or 11.8°. 6. 72°2', or 72.0°. 7. 83°15', 83.3°. 8. 81°50', or 81.8°. 9. 39°28', or 39.3°. 10. 34°11', or 34.2°.

11. 15°38', or 15.6°. 12. 8°32', or 8.5°. 13. 58°48', or 58.8°. 14. 61°0', or 61.0°. 15. 71°5', or 71.1°.

Pages 500-503. 1. $c = 80$. 2. $a = 2.9764$, or 3.0 approx. 3. $a = 7.002$, or 7.0 approx. 4. $a = 28.016$, or 28, approx. 5. $a = 8.9088$, or 8.91 approx. 6. $A = 53°8'$, or 53°10' approx. 7. $B = 48°27'$, or 48° approx. 8. 149.8 ft., or 150 ft, approx. 9. 323.9 ft. 10. 348.3 ft. 11. 39°36', or 40° approx. 12. 4°46', or 5° approx. 13. 11°52', or 11°50' approx. 14. 97°12', or 97° approx. 15. 73°44', or 70° approx. 16. 291.5 ft. 17. 36°52', or 53°8'. 18. 412.8 ft. 19. 101.52 sq.ft., or 100 sq.ft., approx. 20. 13.10 sq.ft. 21. 4.7024 in. 22. 11.50 ft., or 12 ft., approx. 23. 3.464 in., or 3.5 in., approx. 24. 37.088, or 37 approx. 25. 772.8, or 770 ft., approx. 26. 92°7', or 92° approx. 27. 16.91 ft., 17 ft. approx. 32. $x = 8.66$. 33. $y = 37°51'$, or 38° approx.

Page 503 (center). 1. F. 2. T. 3. F. 4. F. 5. T. 6. T. 7. F. 8. T.

Page 503 (foot). 1. $\sin A = .4706$. $\cos A = .8824$. 2. $\tan B = 1.875$. 3. 4.702 in.

Page 505. 2. b and d.

Page 506. 1. $\angle y > \angle s$. 2. $AC > DF$. 6. Reverse order. 7. m is positive. 9. $a^2 > b^2$.

Page 507. 1. $\angle A$; $\angle B$.

Pages 508-509. 1. MP; NP. 2. AC; BC. 6. The side opp. the 30° $\angle$.

Page 511. 1. $AB > DF$.

Page 512. 1. (1) DH. (2) $EF > DF$.

Pages 514-515. 1. Yes. 3. Less than.

Page 516. 2. $\angle A > \angle C > \angle B$.

Pages 516-517. 1. F. 2. T. 3. T. 4. F. 5. F. 6. F. 7. T. 8. F.

Page 517. 1. unequal. 2. greater than. 3. any other chord. 4. chords. 5. increased. 6. greater. 7. less than. 8. hypotenuse. 9. perpendicular. 10. 400.

Page 523. 1. $\frac{3}{5}$; $\frac{1}{2}$; $-\frac{2}{3}$. 2. a. 1. b. -1. c. -1. d. $\frac{4}{7}$. e. 0. f. $\frac{1}{3}$. 4. $x = 4$. 6. AB, $\frac{2}{7}$; DC, $\frac{2}{7}$; CB, $-\frac{3}{2}$; DA, $-\frac{3}{2}$. 7. A parallelogram.

Page 527. 1. a. (5, 2), (2, 4), (2, 0). b. (-3, 3), (1, 2), (2, 2). 2. (-4, -8). 3. 16.

Page 529. 1. a. $4\sqrt{5}$, $\sqrt{34}$, $\sqrt{58}$. b. $\sqrt{74}$, $\sqrt{29}$, $\sqrt{53}$. 6. 5, $\sqrt{85}$.

Page 533. 1. a. $5x - 2y = 4$. b. $x - y = 3$. c. $x + 2y = -6$. d. $y - 7 = 0$. e. $x - 2y = -9$. f. $2x + y = 6$.

Page 534. 1. a. $2x - y = -5$. b. $x - 3y = 9$. c. $5x - y = 2$. d. $x - 2y = -6$. 2. a. $y = -\frac{2}{3}x + 4$. $m = -\frac{2}{3}$, y-intercept $= 4$. b. $y = 5x - 8$. $m = 5$, y-intercept $= -8$. c. $y = x - 6$. $m = 1$, y-intercept $= -6$. 3. Parallel, a and e; perpendicular, c and f.

Page 535. 1. a. $2x + 3y = 17$. b. $2x - y = 7$. c. $x - 2y = 14$. d. $2x + 7y = 21$. e. $5x - 9y = 2$. f. $2x - 7y = 12$. 2. $2x - y = 6$, $x + 5y = 25$, $5x + 3y = -7$. 3. $3x - 7y = 0$, $6x - 5y = 18$, $3x + 2y = 18$. 4. $x = 0$, $2x - 3y + 8 = 0$, $2x + 3y - 8 = 0$. 5. a. $4\sqrt{2}$. b. 24.

Page 536. 1. a. $4x + 3y = 12$. b. $x - 5y = 5$. c. $2x - 5y = -10$. d. $7x - 3y = 21$. 2. a. $\frac{x}{5} + \frac{y}{3} = 1$. b. $\frac{x}{4} + \frac{y}{3} = 1$. c. $\frac{x}{5} - \frac{y}{1} = 1$.

Pages 537-538. 1. a. $x^2 + y^2 = 25$. b. $x^2 + y^2 - 4y - 5 = 0$. c. $x^2 + y^2 + 6x - 7 = 0$. d. $x^2 + y^2 + 2x - 6y + 6 = 0$. e. $x^2 + y^2 - 2x + 8y - 47 = 0$. f. $x^2 + y^2 - 4x - 10y - 7 = 0$. 2. $x^2 + y^2 + 8x - 6y = 0$. 3. $x^2 + y^2 - 4x - 10y + 11 = 0$. 4. a. Center, (0, 0); radius, 10. b. Center, (-5, -4); radius, 4. c. Center, (-5, 12); radius, 13. d. Center, (3, 0); radius, 4.

Page 539 (top). 1. The point (9, 3). 2. The point (10, 4). 3. The point at approx. $(2, 5\frac{1}{2})$. 4. The point (3, -4). 5. The point (-1, -5). 6. The point (3, 2). 7. Two points at approx. (-4, -4) and approx. (4, 4). 8. Two points at (8, 0) and approx. $(-5, 6\frac{1}{2})$. 9. Two points at (3, -3) and (3, 3).

Page 539 (foot). 1. $\sqrt{58}$, $6\sqrt{2}$, $\sqrt{82}$. 2. (2, 0), (6, -1), (1, 5). 3. 1. 6. $2x - 3y = -11$. 7. $2x - y = -5$. 8. $3x + 2y = 19$. 9. $4x - 3y = 12$. 10. $x^2 + y^2 = 36$. 11. $x^2 + y^2 - 8x + 6y = 0$. 12. $x^2 + y^2 - 12x - 4y + 24 = 0$.

Page 542. 1. 106 lb. 2. 5 lb. in direction of AB. 3. (1) 94 lb. (2) AD is 94 lb. (3) $\angle$ BAD = 152°.

Pages 543-544. 1. 26,410 lb. 2. b. 111.8 m.p.h. 3. 360 miles.

Page 546. 1. a. 0418. b. 1200. c. 1815. d. 2332. e. 0700. f. 2400, or 0000. 2. 0119, or 1:19 A.M., Thursday. 3. 2055, or 8:55 P.M., Monday. 4. a. 45°56'26". b. 8°40'42". 5. 232°18'. 6. 317°. 7. Zero degrees. 8. 148 mi.

Page 548. 1. 42 mi., approx. 2. RT, 195 mi., approx.; ST, 96 mi., approx. 3. 183 mi. from x, approx.

Page 551. 1. Course, 45°, approx. Ground speed, 150 m.p.h., approx. Drift angle, 15°, approx. 2. Course, 12°, approx. Ground speed, 139 m.p.h., approx. Drift angle, 18°, approx. 3. Course, 51°, approx. Ground speed, 98 m.p.h., approx. Drift angle, 21°, approx. 4. 304 m.p.h., approx. 5. (Wind, 20 m.p.h.). 288 m.p.h., approx.

Page 552. 1. 30°, 106 m.p.h., approx. 2. 271°, approx., 225 m.p.h., approx. 3. 198 m.p.h., approx., 278°, approx.

Pages 554-555. 1. (1) $45\frac{1}{2}^{\circ}$, approx. (2) 11.4 min., approx. 2. 13.9 min., approx. 3. (1) 187 mi., approx. (2) 77 mi., approx. (3) 1 hr. $2\frac{1}{2}$ min., approx. 4. 233°, approx., 19.6 min approx. 5. (1) 346°, approx. (2) 84 mi., approx. (3) 1 hr. 12 min., approx. 6. (1) 1°, approx. (2) 2.4 ft., approx.

Page 556. 1. 360 mi. 2. (1) 135 mi., approx. (2) 109°, approx. 3. $212\frac{8}{11}$ mi.

Pages 557-558. 1. I. 2. III. 3. I. 4. II. 5. I. 6. III. 7. II.

Page 559. 1. a. 6. b. 4. c. 1. d. 0. e. 1 f. 0. 2. a. 16. b. 4. c. 4. d. 3. e. 4. f. 1. 3. a. 27. b. 0. c. 0. d. 8. e. 12. f. 6. 4. a. 24. b. 5. c. 12. d. 5. e. 2. f. 0. 5. a. 32. b. 2. c. 9. d. 18. e. 3. f. 0. 6. a. 10. b. 4. c. 4. d. 1. e. 0. f. 1.

ANSWERS

TO ACHIEVEMENT TESTS

TEST 1 (CHAPTER 1)

Part 1. Completing Statements

1. equal. 2. rays. 3. coincide. 4. $\perp$. 5. bisected. 6. oblique. 7. acute. 8. complementary. 9. 142°50'. 10. arc. 11. perpendicular. 12. adjacent. 13. vertical. 14. $\angle n$. 15. DE. 16. $\angle n$. 17. perpendicular. 18. straight. 19. radius. 20. perpendicular.

Part 2. Applications

21. 49°. 22. $(80 - x)^\circ$. 23. 69°20'. 24. 1125'. 25. 75°. 26. 108°. 27. 71°46'40". 28. 33°. 29. 60°. 30. 100°.

TEST 2 (CHAPTER 2)

1. Inductive reasoning. 2. The general principle is false. 3. CD = $\frac{1}{2}$AB, regardless of the lengths of CA and CB which are perpendicular to each other at C. 4. (1) FE seems to be $\frac{1}{2}$ the sum of AB and DC. (2) MN is not equal to $\frac{1}{2}$ the sum of GH and LK. 5. It appears that the sum of the sides of DEF is one half the sum of the sides of ABC.

TEST 3 (CHAPTER 3)

Part 1. True-False Statements

1. False. 2. True. 3. True. 4. True. 5. True. 6. False. 7. True. 8. False. 9. True. 10. True.

Part 2. Supplying Reasons

11. §20. 12. Asmt. 1. 13. Asmt. 4. 14. Asmt. 6 or Asmt. 7. 15. Asmt. 26. 16. Asmt. 3. 17. Asmt. 8. 18. §22. 19. Asmt. 21. 20. Asmt. 18. 21. §13. 22. §22. 23. Asmt. 9. 24. Asmt. 2. 25. Asmt. 24. 26. Asmt. 24. 27. Asmt. 31. 28. §14. 29. §20. 30. Asmt. 28.

TEST 4 (CHAPTER 4)

Part 1. True-False Statements

1. False. 2. False. 3. True. 4. True. 5. True. 6. False. 7. False. 8. False. 9. True. 10. True.

Part 2. Completing Statements

11. two. 12. perimeter. 13. hypotenuse. 14. median.
15. $\angle B$. 16. 50. 17. 50. 18. 90. 19. 35. 20. BC.

Part 3. Determining Hypothesis and Conclusion

21. If two altitudes of a triangle are equal, the triangle is isosceles.

22. Two isosceles triangles are congruent if the base and vertex angle of one are equal respectively to the base and vertex angle of the other.

23. The line segment joining the midpoints of two sides of a triangle is one half the third side.

24. If a point is on the bisector of an angle, it is equidistant from the sides of the angle.

25. Two right triangles are congruent if the hypotenuse and a leg of one are equal respectively to the hypotenuse and a leg of the other.

Part 4. Supplying Reasons

26. Given. 27. § 22. 28. § 49. 29. Asmt. 32.
30. Asmt. 34. 31. Given. 32. Given. 33. § 22. 34. Iden.
35. § 85. 36. Asmt. 34. 37. § 14. 38. § 90. 39. Asmt. 34.
40. § 22.

Part 5. Proofs

41. Proof: 1. ABCD is a polygon. (Given.) 2. ABC and ADC are $\triangle$s. (§ 55.) 3. AB = AD. (Given.) 4. BC = DC. (Given.) 5. AC = AC. (Iden.) 6. $\triangle ABC \cong \triangle ADC$. (§ 85.) 7. $\angle ABC = \angle ADC$. (Asmt. 34.)

42. Proof: 1. In $\triangle ABC$, AC = BC. (Given.) 2. AD = EB. (Given.) 3. $\angle A = \angle B$. (§ 83.) 4. $\triangle ADC \cong \triangle BEC$. (Asmt. 32.) 5. CD = CE. (Asmt. 34.)

TEST 5 (CHAPTER 5)

Part 1. Completing Statements

1. complementary. 2. 180. 3. supplementary. 4. intersect. 5. corresponding. 6. m. 7. vertical. 8. 82.
9. 98. 10. 98. 11. 8. 12. 35. 13. BE. 14. AEC.
15. central. 16. polygon. 17. D. 18. DCE. 19. E.
20. 60.

Part 2. Supplying Reasons

21. §111. 22. §113. 23. §112. 24. §117. 25. §108.
26. Asmt. 23. 27. §112. 28. §112. 29. §117. 30. Asmt. 6.

Part 3. Multiple-Choice

31. d. 32. c. 33. d. 34. b. 35. a. 36. b.
37. c. 38. b. 39. b. 40. a.

Part 4. Proofs

41. 1. In △MNP, MP = NP. (Given.) 2. PQ is the altitude of △MNP. (Given.) 3. PQ ⊥ MN. (§90.) 4. ∠MQP and ∠NQP are rt. ∠s. (§14.) 5. MQP and NQP are rt. △s. (§57.) 6. ∠M = ∠N. (§83.) 7. △MQP ≅ △QNP. (§125.) 8. ∠x = ∠y. (Asmt. 34.)

42. Proof: 1. In polygon ABCD, AB ∥ CD. (Given.) 2. AD = DC. (Given.) 3. ∠y = ∠x. (§83.) 3. ∠z = ∠x. (§111.) 4. ∠y = ∠z. (Asmt. 7.) 5. AC bisects ∠BAD. (§22.)

43. Proof: 1. OA and OB are radii of ⊙O. (Given.) 2. OA = OB. (Asmt. 26.) 3. AC = CB. (Given.) 4. OC bisects AB. (§140.) 5. AE = EB. (§22.)

Part 5. Converses, Inverses, Contrapositives

44. If two angles of a triangle are equal, the sides opposite these angles are equal. Yes.

45. If two sides of a triangle are unequal, the angles opposite these sides are unequal. The inverse is true because the converse is true.

46. If two angles of a triangle are unequal, the sides opposite these angles are unequal. The contrapositive of the theorem is true because the theorem is true.

47. (1) Given △ABC, CD ⊥ AB and AC ≠ BC.
Prove AD ≠ DB.
(2) Given △ABC, AC ≠ BC, and AD = DB.
Prove CD not ⊥ AB.

TEST 6 (CUMULATIVE. CHAPTERS 1-5)

Form A

Part 1. Completing Statements

1. congruent. 2. exterior. 3. altitude. 4. 66.
5. median. 6. theorem. 7. conclusion. 8. 65°15'.
9. 79°30'. 10. A.

Part 2. Supplying Reasons

11. Given. 12. §83. 13. Given. 14. §14. 15. §57.
16. §124. 17. Asmt. 29. 18. §49. 19. Asmt. 7. 20. §128.

Part 3. Proofs

21. **Proof**: 1. In $\triangle RWT$, $\angle a = \angle b$. (Given.) 2. $RS \perp WT$. (Given.) 3. $\angle WSR = \angle TSR$. (§14.) 4. $RS = RS$. (Iden.) 5. $\triangle WRS \cong \triangle TRS$. (Asmt. 33.) 6. $\angle W = \angle T$. (Asmt. 34.)

22. **Proof**: 1. In $\triangle MNT$, $MT = NT$. (Given.) 2. $\angle M = \angle N$. (§83.) 3. $MR = SN$. (Given.) 4. $RS = RS$. (Iden.) 5. $MR + RS = RS + SN$. (Asmt. 1.) 6. $MR + RS = MS$ and $RS + SN = RN$. (Asmt. 9.) 7. $MS = RN$. (Asmt. 6, or 7.) 8. $\triangle MTS \cong \triangle RTN$. (Asmt. 32.)

23. **Proof**: 1. In polygon FGHK, $KH \parallel FG$. (Given.) 2. AB and KG are st. lines. (Given.) 3. O is the midpoint of KG. (Given.) 4. $KO = OG$. (§22.) 5. $\angle AKO = \angle BGO$. (§111.) 6. $\angle KOA = \angle BOG$. (§49). 7. $\triangle AOK \cong \triangle BOG$. (Asmt. 33.)

24. **Proof**: 1. In $\triangle ABC$, AC is produced to E so that $CE = BC$. (Given.) 2. $CD \parallel EB$. (Given.) 3. From (1), $\angle E = \angle CBE$. (§83.) 4. From (2), $\angle x = \angle E$. (§112.) 5. From (2), $\angle y = \angle CBE$. (§111.) 6. Then $\angle x = \angle y$. (Asmt. 7.)

TEST 6 (CUMULATIVE. CHAPTERS 1-5)

Form B

Part 1. Completing Statements

1. supplementary. 2. 72. 3. 135. 4. assumptions.
5. parallel. 6. inverse. 7. equiangular. 8. adjacent.
9. hypothesis. 10. right.

Part 2. Supplying Reasons

11. Given. 12. §124. 13. §20. 14. §22. 15. Asmt. 6.
16. Asmt. 1. 17. Asmt. 9. 18. Asmt. 6. 19. §21. 20. §106.

Part 3. Proofs

21. **Proof**: 1. $\angle A = \angle B$. (Given.) 2. $DE \parallel AB$. (Given.) 3. $\angle CDE = \angle A$. (§112.) 4. $\angle CED = \angle B$. (§112.) 5. $\angle CDE = \angle CED$. (Asmt. 7.) 6. $CD = CE$. (§128.) 7. $AC = BC$. (§128.) 8. $AD = BE$. (Asmt. 2.)

22. **Proof**: 1. ACB is a rt. $\angle$. (Given.) 2. $CD \perp AB$. (Given.) 3. $\angle ADC$ is a rt. $\angle$. (§14.) 4. $\angle ACB = \angle ADC$. (Asmt. 19.) 5. ADC is a rt. $\triangle$. (§57.) 6. $\angle A = \angle A$. (Iden.) 7. $\angle x = \angle B$. (§121.)

23. Proof: 1. In △YXZ, ∠r = ∠s. (Given.) 2. XW ⊥ YZ. (Given.) 3. ∠YWX = ∠ZWX. (§14.) 4. XW = XW. (Iden.) 5. △YWX ≅ △ZWX. (Asmt. 33.) 6. YW = WZ. (Asmt. 34.)

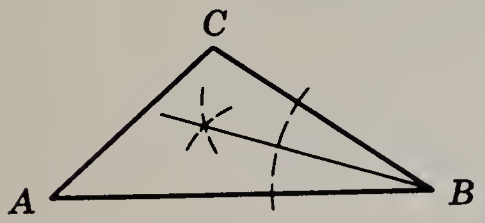
24. Proof: 1. AEFB is a st. line. (Given.) 2. ∠A = ∠B. (Given.) 3. AC = BC. (§128.) 4. AE = FB. (Given.) 5. EF = EF. (Iden.) 6. AE + EF = EF + FB. (Asmt. 1.) 7. AE + EF = AF and EF + FB = EB. (Asmt. 9.) 8. AF = EB. (Asmt. 6.) 9. △AFC ≅ △EBC. (Asmt. 32.)

25. Proof: 1. △APB is isosceles having ∠APB the vertex angle. (Given.) 2. AP = PB. (§§56, 58.) 3. △AQB is isosceles having AQB the vertex angle. (Given.) 4. AQ = BQ. (§§56, 58.) 5. PQ = PQ. (Iden.) 6. △APQ ≅ △BPQ. (§85.) 7. ∠APQ = ∠QPB. (Asmt. 34.) 8. PQ bisects ∠APB. (§22.)

TEST 7 (CHAPTER 6)

Constructions

1.



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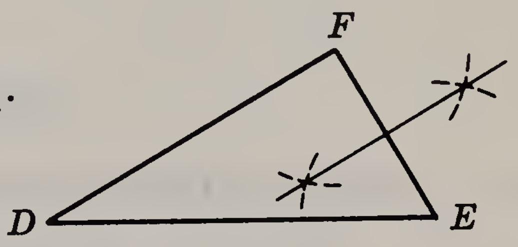

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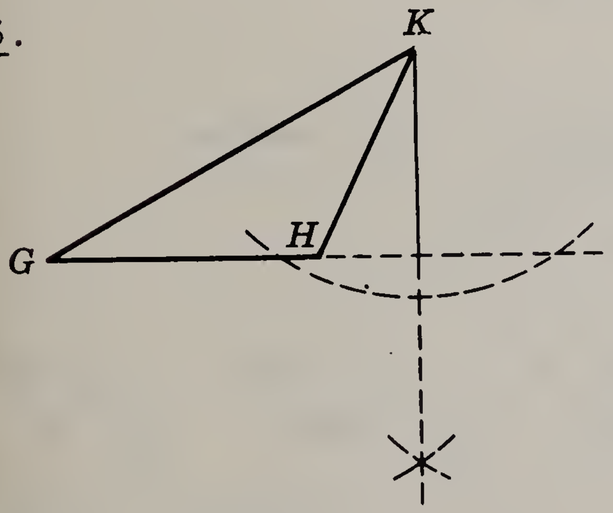

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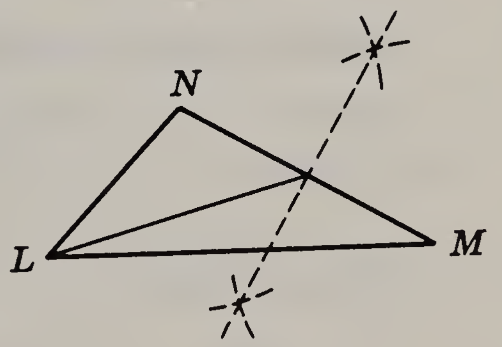

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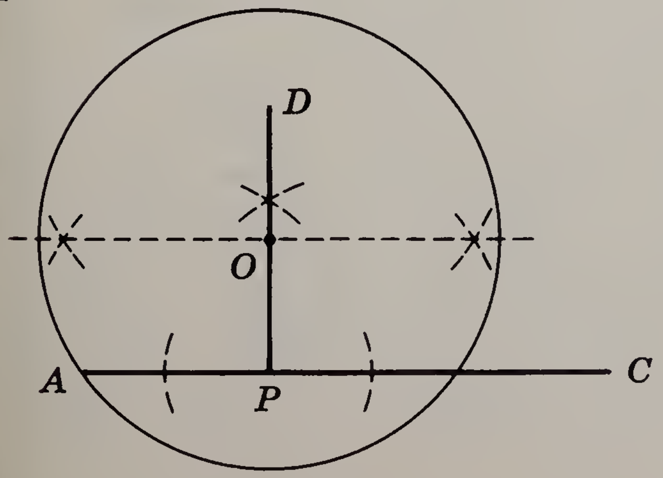

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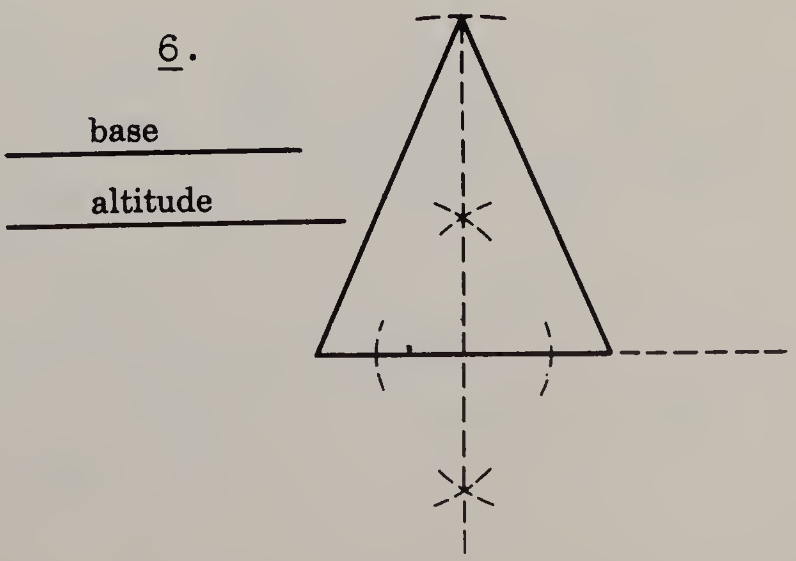

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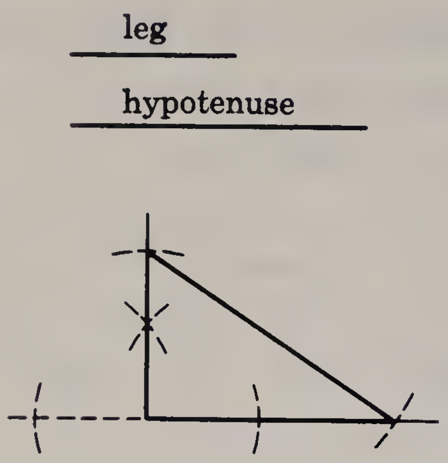

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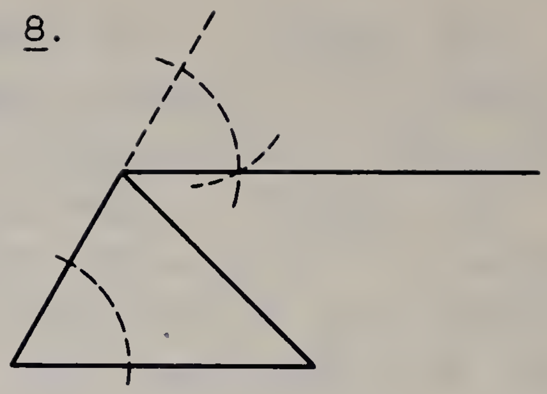

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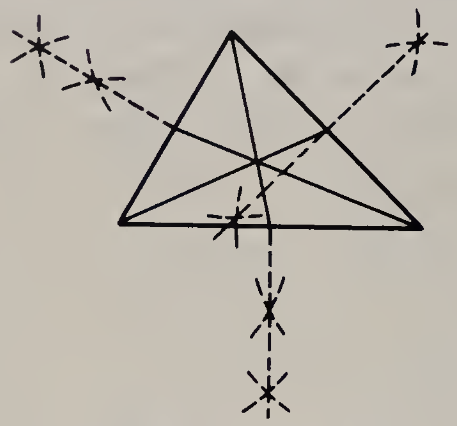

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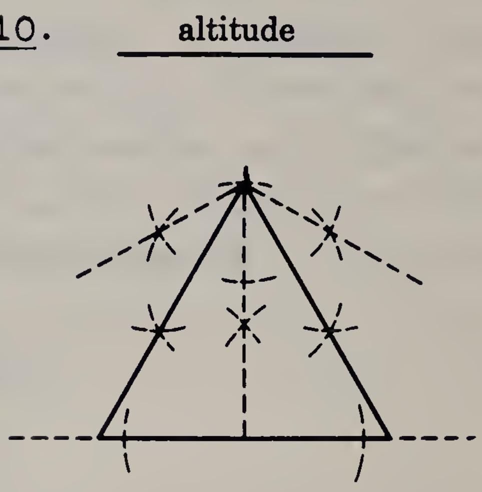


TEST 8 (CHAPTER 7)

Part 1. Multiple-Choice Statements

1. b. 2. c. 3. c. 4. b. 5. b.

Part 2. Completing Statements

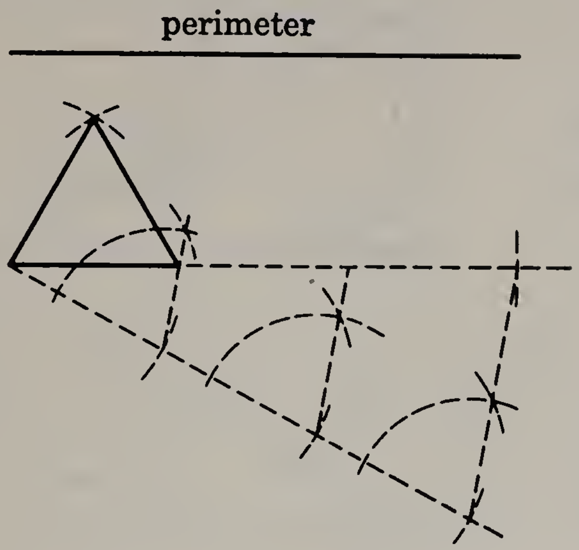
6. square. 7. trapezoid. 8. square. 9. median. 10. quadrilateral.

Part 3. Applications

11. 4 rt. ∠s. 12. 1800°, or 10 st. ∠s. 13. 15. 14. 18. 15. 52. 16. 9 inches. 17. 20 inches. 18. 144°. 19. 8 inches. 20. 5.

Part 4. Constructions

21.



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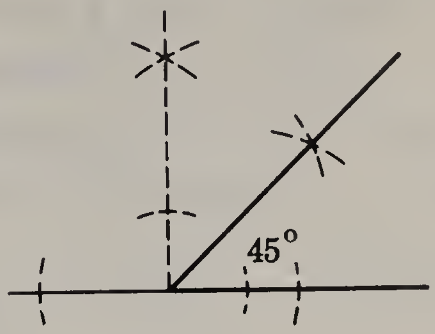

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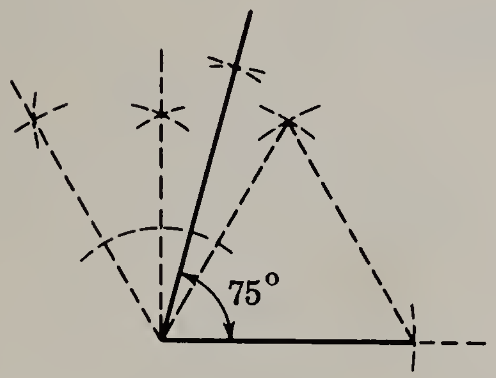

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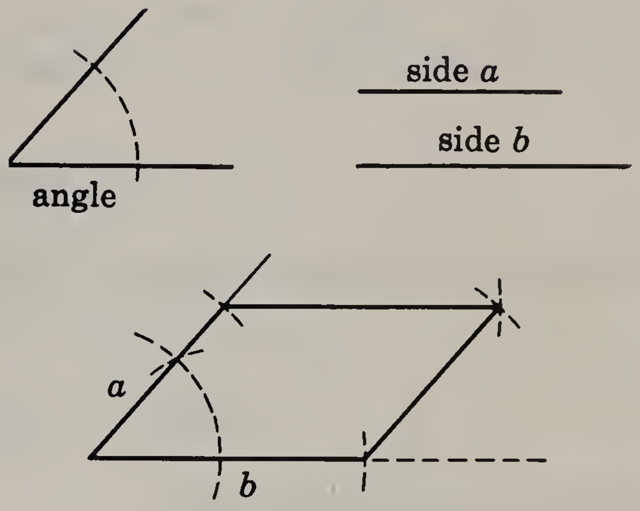

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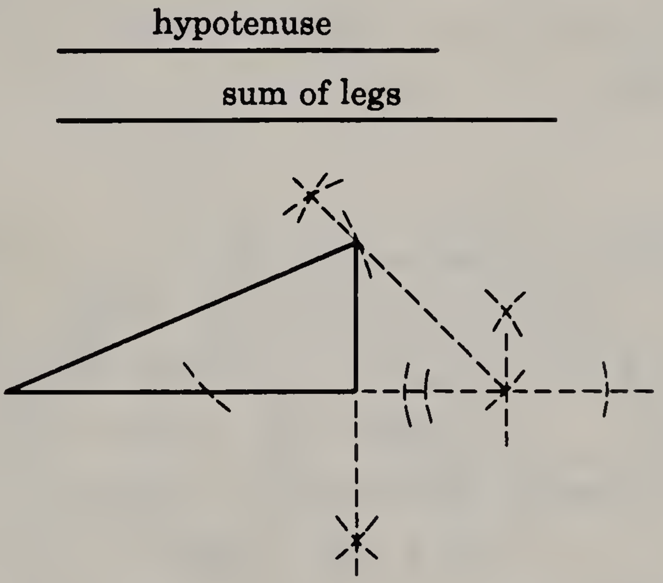
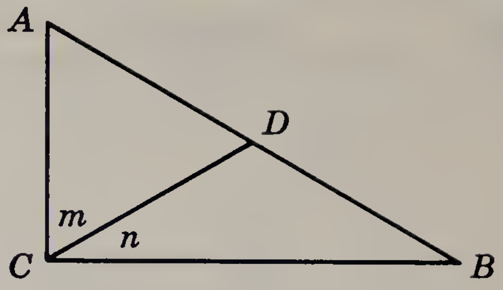


Part 5. Proofs.

Proof: 26. ABCD is a ▱. (Given.) 27. AB ∥ DC. (§185.) 28. ∠ n = ∠ x. (§111.) 29. AC bisects ∠ BCD. (Given.) 30. ∠ y = ∠ x. (§22.) 31. ∠ n = ∠ y. (Asmt. 7.) 32. AB = BC. (§128.) 33. Then ABCD is a rhombus. (§185.)

34. Proof: 1. ABCD is a ▱. (Given.) 2. AD = BC, AB = DC. (§188.) 3. DF bisects AB and BE bisects DC. (Given.) 4. Then AF = EC. (Asmt. 4.) 5. ∠ A = ∠ C. (§189.) 6. △ AFD ≅ △ BEC. (Asmt. 32.) 7. Then DF = BE. (Asmt. 34.)

35. Given △ ABC, having median CD = $\frac{1}{2}$AB. Prove that ABC is a rt. △. Proof: 1. Median CD = $\frac{1}{2}$AB. (Given.) 2. AD = $\frac{1}{2}$AB and BD = $\frac{1}{2}$AB. (§89.) 3. Then CD = AD and CD = BD. (Asmt. 7.) 4. ∠n = ∠B and ∠m = ∠A. (§83.) 5. ∠A + ∠ACB + ∠B = 1 st. ∠, or 2 rt. ∠s. (§120.) 6. ∠ACB = ∠m + ∠n. (Asmt. 9.) 7. Then ∠m + ∠m + ∠n + ∠n = 2 rt. ∠s. (Asmt. 6.) 8. 2∠m + 2∠n = 2 rt. ∠s. (Asmt. 6.) 9. ∠m + ∠n = 1 rt. ∠. (Asmt. 4.) 10. ∠ACB = 1 rt. ∠. (Asmts. 9, 6.) 11. ABC is a rt. △. (§57.)



TEST 9 (CHAPTER 8)

Part 1. Multiple-Choice Statements

1. a. 2. d. 3. c. 4. c. 5. b. 6. a. 7. d. 8. c. 9. a. 10. b.

Part 2. Applications

11. 65. 12. 9 : 16. 13. $15\sqrt{2}$ ft. 14. 121.5 sq.in. 15. 98 in. 16. 98 sq.in. 17. 56 sq.in. 18. 12 in. 19. 270. 20. $9\frac{3}{13}$.

Part 3. Proofs

21. See page 243 of the textbook.

22. Proof: 1. Draw DE ⊥ AB and CG ⊥ AB. (Asmt. 20.) 2. DE is alt. of △ ABD and CG is alt. of △ ABC. (§90.) 3. △ ABD = $\frac{1}{2}$ x AB x DE. (§238.) 4. △ ABC = $\frac{1}{2}$ x AB x CG. (§238.) 5. ABCD is a trapezoid with bases AB and DC. (Given.) 6. DC ∥ AB. (§206.) 7. DE ∥ CG. (§107.) 8. DE = CG. (§192.) 9. From (4) and (8), △ABC = $\frac{1}{2}$ x AB x DE. (Asmt. 6.) 10. From (3) and (9), △ABD = △ABC. (Asmt. 7.) 11. △ ABD = △ ABF + △ AFD and △ ABC = △ ABF + △BFC. (Asmt. 9.) 12. △ ABF + △ AFD = △ ABF + △ BFC. (Asmt. 6.) 13. △ AFD = △ BFC. (Asmt. 2.)

23. Asmt. 32.

24. § 243.

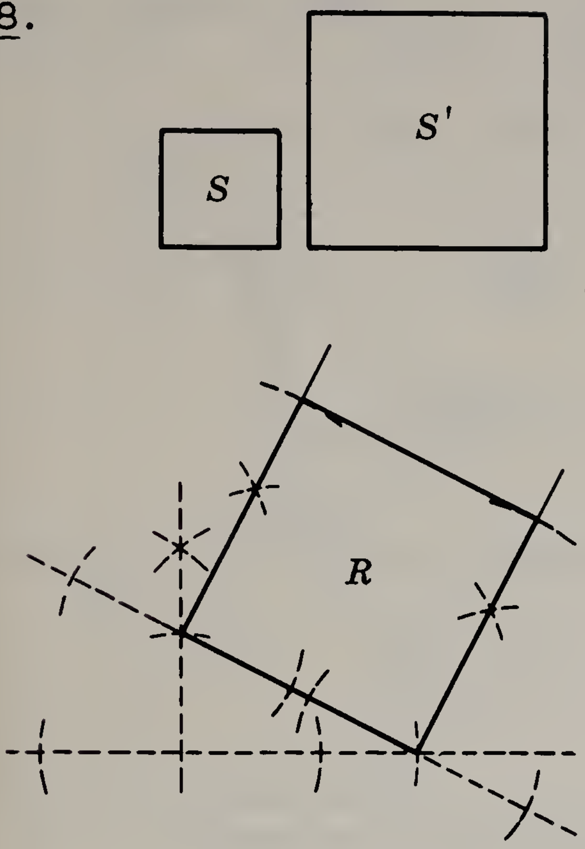
25. § 243.

26. It is necessary to show that LKC is a straight line in order to prove S and △ ABC have equal altitudes to AB.

27. Asmt. 3.

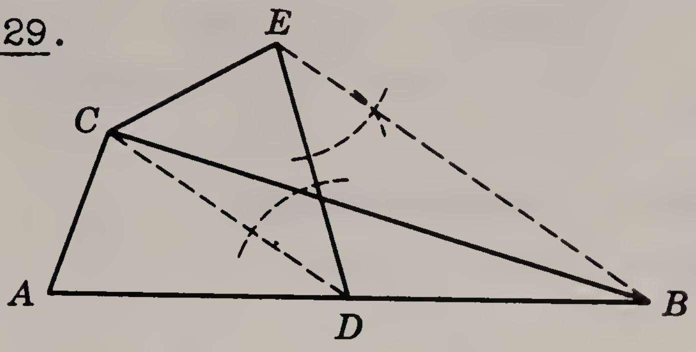
Part 4. Constructions

28.



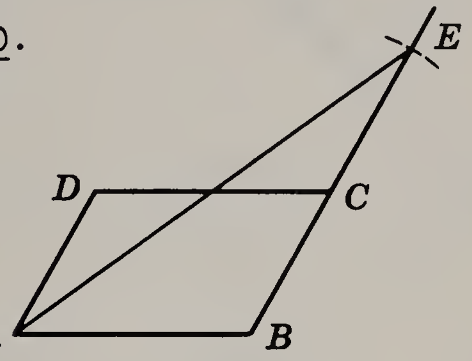
R = S + S'

29.



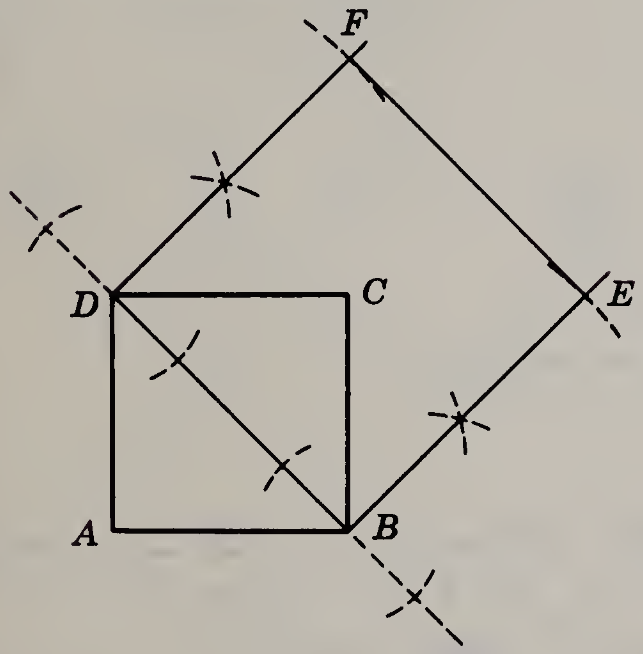
△ABC = quad. ADEC

30.

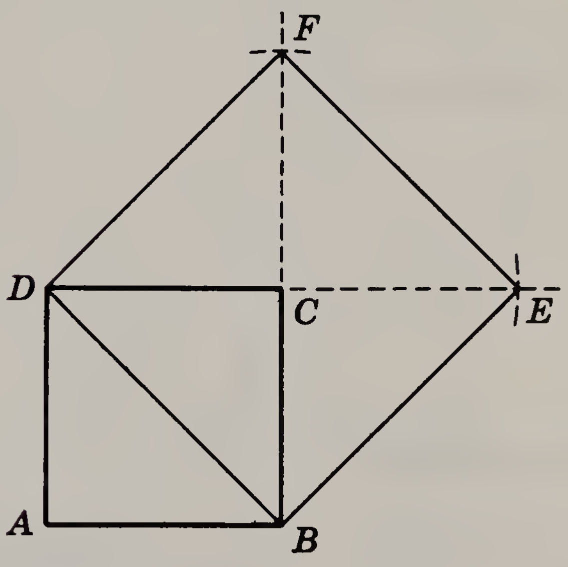


△ABE = ▱ ABCD

31.



Solution 1



Solution 2

TEST 10 (CUMULATIVE. CHAPTERS 1-8)

Form A

Part 1. True-False Statements

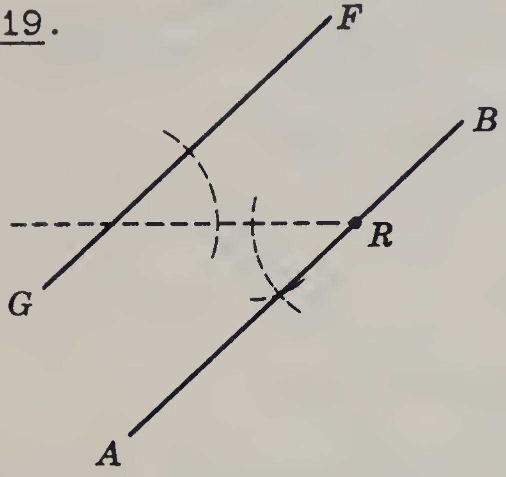
1. False. 2. True. 3. False. 4. True. 5. True.
6. True. 7. False. 8. False. 9. True. 10. True.

Part 2. Numerical

11. 135. 12. 20. 13. 10 inches. 14. 7.5. 15. 74°.
16. 120 sq.in. 17. 30 feet. 18. 91.

Part 3. Constructions

19.



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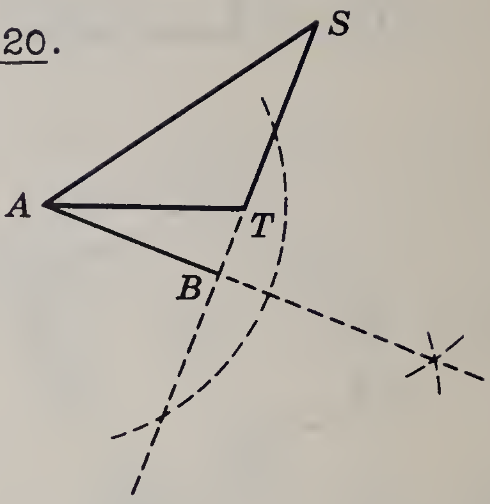

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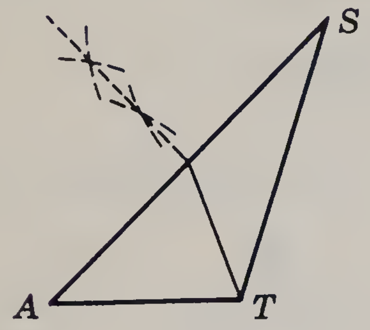

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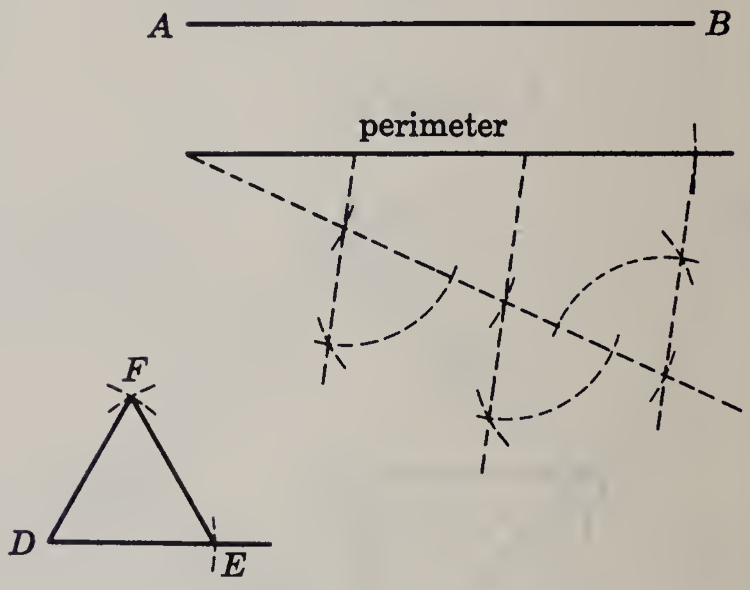


Part 4. Proofs

23. Proof: 1. In △ABC, AC = BC. (Given.) 2. CF is a median. (Given.) 3. AF = FB. (§§89, 22.) 4. CF ⊥ AB. (§140.)

24. Proof: 1. h is an altitude of △DEF. (Given.) 2. h is an altitude of △DKF and △KEF. (§90.) 3. FK is a median of △DEF. (Given.) 4. DK = KE. (§§89, 22.) 5. △DKF = $\frac{1}{2}h \times \overline{DK}$. (§238.) 6. △DKF = $\frac{1}{2}h \times \overline{KE}$. (Asmt. 6.) 7. △KEF = $\frac{1}{2}h \times \overline{KE}$. (§238.) 8. △DKF = △KEF. (Asmt. 7.)

25. Proof: 1. EF bisects AD and BD of △ABD. (Given.) 2. EF ∥ AB. (§204.) 3. HK bisects AC and BC of △ABC. (Given.) 4. HK ∥ AB. (§204.) 5. EF ∥ AB. (§109.)

26. Proof: 1. AC is a diagonal of square ABCD. (Given.) 2. Draw PD. (Asmt. 11.) 3. AP = AD. (Given.) 4. PE ⊥ AC. (Given.) 5. ∠ADC is a right ∠. (§§185, 191.) 6. ∠EPA is a

rt. $\angle$. (§14.) 7. From (3), $\angle$ADP = $\angle$APD. (§83.) 8. $\angle$m = $\angle$n. (Asmt. 29.) 9. DE = PE. (§128.) 10. $\angle$EPC is a rt. $\angle$. (§14.) 11. $\angle$PCE of rt. $\triangle$ PCE = $\angle$ ACD of rt. $\triangle$ ACD. (Iden.) 12. $\angle$ ADC = $\angle$ EPC. (Asmt. 19.) 13. Then $\angle$ x = $\angle$ DAC. (§121.) 14. ABCD is a rhombus. (§185.) 15. AD = DC. (§194.) 16. $\angle$ DAC = $\angle$ DCA. (§83.) 17. $\angle$ x = $\angle$DCA. (Asmt. 6.) 18. PE = PC. (§128
19. Then DE = PC. (Asmt. 6.)

TEST 10 (CUMULATIVE. CHAPTERS 1-8)

Form B

Part 1. True-False Statements

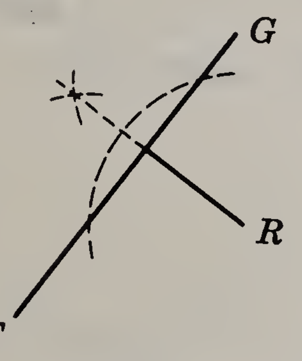
1. True. 2. True. 3. False. 4. True. 5. True. 6. False. 7. False. 8. False. 9. True. 10. True.

Part 2. Numerical

11. 24 in. 12. 102 sq.in. 13. 108. 14. 15. 15. 12 in. 16. 5. 17. 71°. 18. 4 in.

Part 3. Constructions

19.



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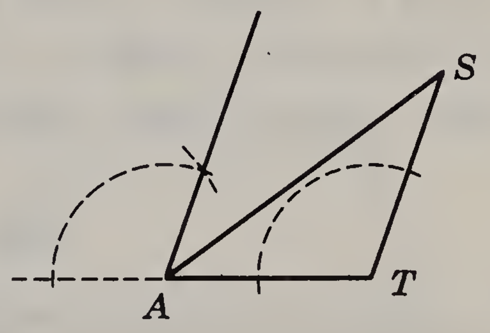

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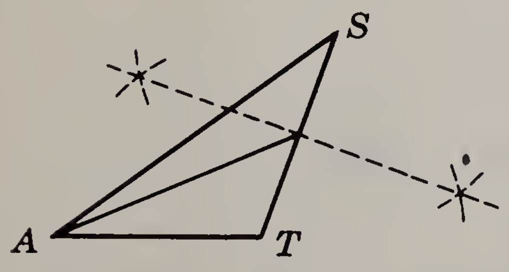

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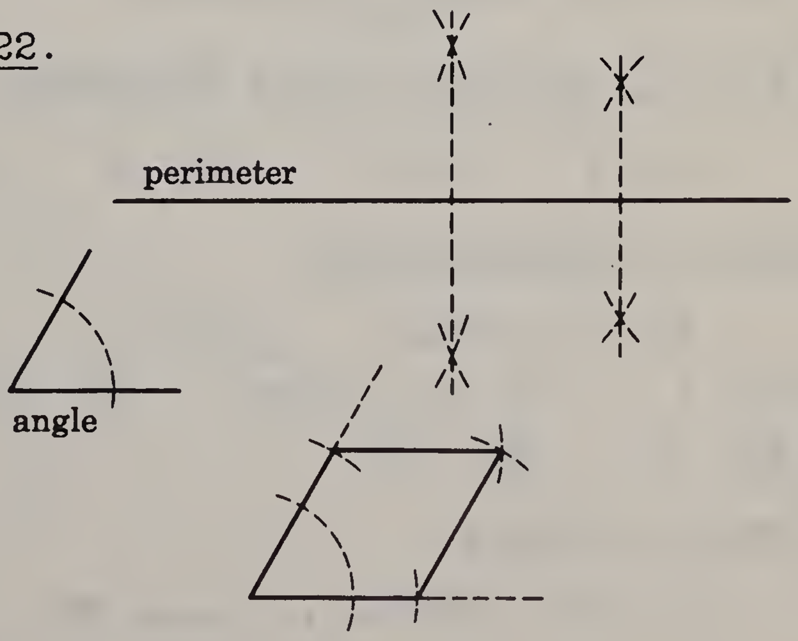


Part 4. Proofs

23. Proof. 1. ABCD is a trapezoid having bases AB and CD. (Given.) 2. AB $\parallel$ CD. (§206.) 3. $\angle m = \angle y$. (§111.) 4. $\angle x = \angle y$. (Given.) 5. $\angle x = \angle m$. (Asmt. 7.) 6. AD = DC. (§128.)

24. Proof: 1. DEF is a triangle with DF = EF. (Given.) 2. HK is the perpendicular bisector of DE. (Given.) 3. Then F lies on HK. (§139.)

25. Proof: 1. In quadrilateral ABCD, AB = AD. (Given.) 2. $\angle ABD = \angle ADB$. (§83.) 3. $\angle ABC = \angle ADC$. (Given.) 4. $\angle ABC = \angle ABD + \angle DBC$ and $\angle ADC = \angle ADB + \angle BDC$. (Asmt. 9.) 5. $\angle ABD + \angle DBC = \angle ADB + \angle BDC$. (Asmt. 6.) 6. $\angle DBC = \angle BDC$. (Asmt. 2.) 7. BC = DC. (§128.)

26. Proof: 1. From F draw line segment $h \perp GE$ and from D draw line segment $a \perp GE$. (Asmt. 20.) 2. h is the altitude of $\triangle GFM$ and of $\triangle EFM$. (§90.) 3. Base GM of $\triangle GMF$ = base ME of $\triangle MEF$. (Given.) 4. $\frac{\triangle GMF}{\triangle EMF} = \frac{h}{h}$, or 1. (§239.) 5. $\triangle GMF = \triangle EMF$. (Asmt. 3.) 6. Base GM of $\triangle GMD$ = base ME of $\triangle EMD$. (Given.) 7. a is the altitude of $\triangle GMD$ and of $\triangle EMD$. (§90.) 8. $\frac{\triangle DMG}{\triangle DME} = \frac{a}{a}$, or 1. (§239.) 9. $\triangle DMG = \triangle DME$. (Asmt. 3.) 10. $\triangle GMF + \triangle DMG = \triangle EMF + \triangle DME$. (Asmt. 1.) 11. $\triangle GMF + \triangle DMG$ = quad. DMFG and $\triangle EMF + \triangle DME$ = quad. DEFM. (Asmt. 9.) 12. quad. DMFG = quad. DEFM. (Asmt. 6.)

TEST 11 (CHAPTER 9)

Part 1. Completing Statements

1. chord. 2. radius. 3. CD. 4. internally. 5. tangents. 6. central. 7. inscribed. 8. chords. 9. internal. 10. either radius or diameter.

Part 2. Multiple-Choice Statements

11. b. 12. b. 13. d. 14. c. 15. c.

Part 3. Applications

16. 17 inches. 17. 8 in. 18. $\frac{40}{3}\sqrt{3}$. 19. 4 inches. 20. 6. 21. 20 inches. 22. $2\sqrt{2}$ inches. 23. 15 inches. 24. 16. 25. $8\sqrt{3}$.

Part 4. Proofs

26. See Theorem 40, page 295, in textbook.

27. Proof: 1. AC is a common external tangent to ⊙ O and ⊙ O'. (Given.) 2. PB is a common internal tangent to both circles. (Given.) 3. PA = PB and PC = PB. (§300). 4. Then AP = PC. (Asmt. 7.)

28. Proof: 1. Draw DO. (Asmt. 11.) 2. $\triangle ABC$ is equilateral. (Given.) 3. $\angle A = \angle B$. (§84.) 4. OD = OB. (Asmt. 26.) 5. $\angle ODB = \angle B$. (§83.) 6. $\angle A = \angle ODB$. (Asmt. 7.) 7. $AC \parallel DO$. (§105.) 8. CO = OB. (Asmt. 26.) 9. AD = DB. (§203.) 10. Then ⊙O bisects AB. (§22.)

TEST 12 (CHAPTER 10)

Part 1. Completing Statements

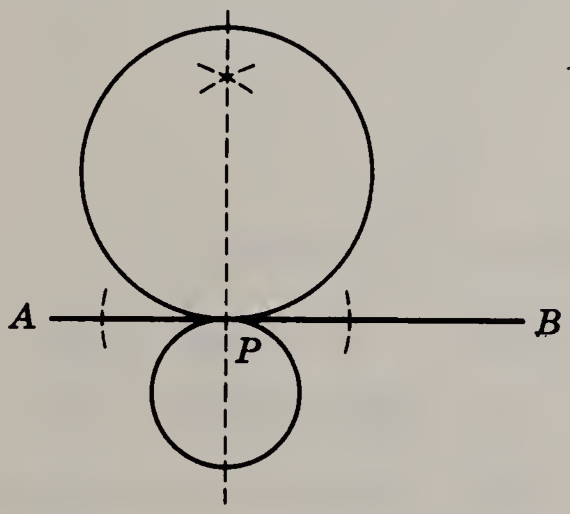
1. tangents. 2. parallel. 3. supplementary. 4. 6°. 5. obtuse. 6. acute. 7. rhombus. 8. half. 9. equal.

Part 2. Applications

10. 60°. 11. 140°. 12. 45°. 13. 35°. 14. 140°. 15. 36°. 16. 78°. 17. 66°. 18. 30°. 19. $10\sqrt{2}$.

Part 3. Constructions

20.



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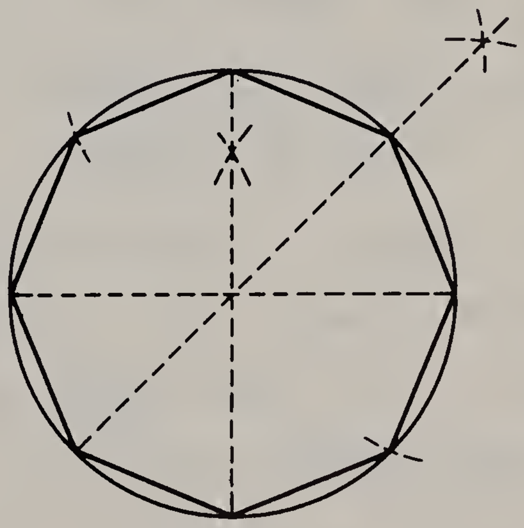

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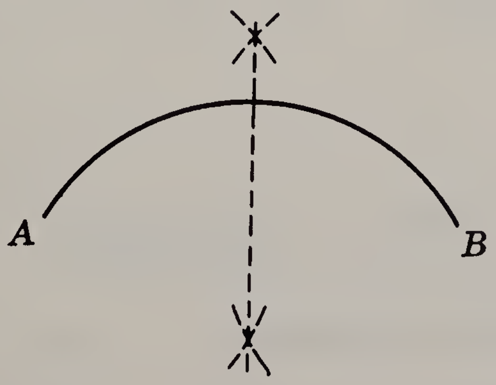

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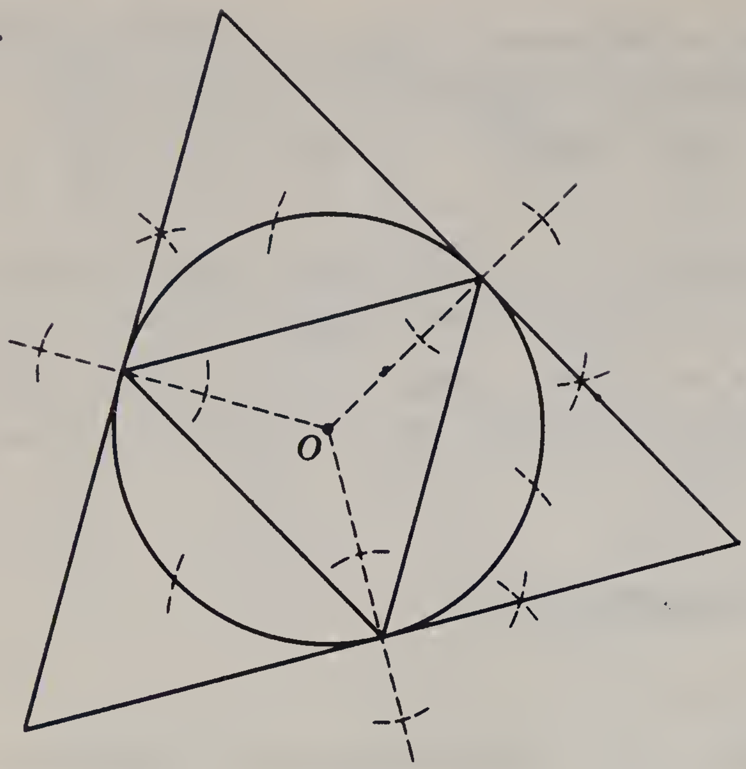

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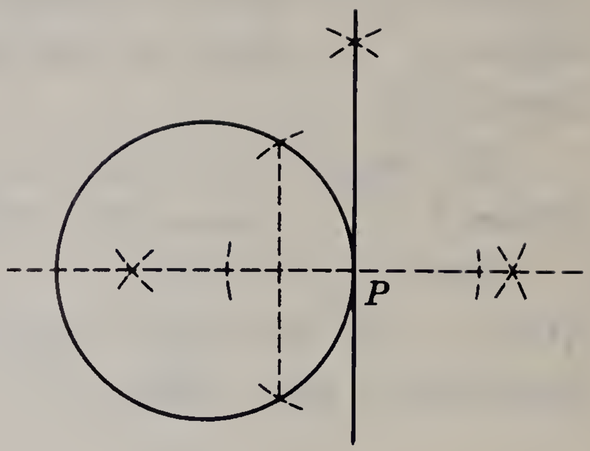


Part 4. Proofs

25. Draw HF tangent to $\odot O$ at P. (§329.)

26. HF is tangent to $\odot O'$. (§302.)

27. $\angle BCP \overset{\circ}{=} \frac{1}{2}\widehat{BP}$ and $\angle E \overset{\circ}{=} \frac{1}{2}\widehat{DP}$. (§318.)

28. $\angle DPH \overset{\circ}{=} \frac{1}{2}\widehat{DP}$ and $\angle BPH \overset{\circ}{=} \frac{1}{2}\widehat{BP}$. (§322.)

29. $\frac{1}{2}\widehat{DP} \overset{\circ}{=} \frac{1}{2}\widehat{BP}$. (Asmt. 7.)

30. $\angle BCP = \angle E$. (Asmt. 7.)

31. $\therefore BC \parallel DE$. (§105.)

32. Proof: 1. Radius $PO \perp$ chord AB of $\odot O$. (Given.). 2. Draw chord BP. (Asmt. 11.) 3. $\widehat{AP} = \widehat{BP}$. (§290.) 4. $\angle ABP \overset{\circ}{=} \frac{1}{2}\widehat{AP}$. (§318.) 5. $\angle ABP \overset{\circ}{=} \frac{1}{2}\widehat{BP}$. (Asmt. 6.) 6. $\angle PBD \overset{\circ}{=} \frac{1}{2}\widehat{BP}$. (§322.) 7. $\angle ABP = \angle PBD$. (Asmt. 7.) 8. $PD \perp BD$. (Given.) 9. PBC and PBD are rt. $\triangle$s. (§57.) 10. Hyp. PB = Hyp. PB. (Iden.) 11. $\triangle PBC \cong \triangle PBD$. (§125.) 12. PC = PD. (Asmt. 34.)

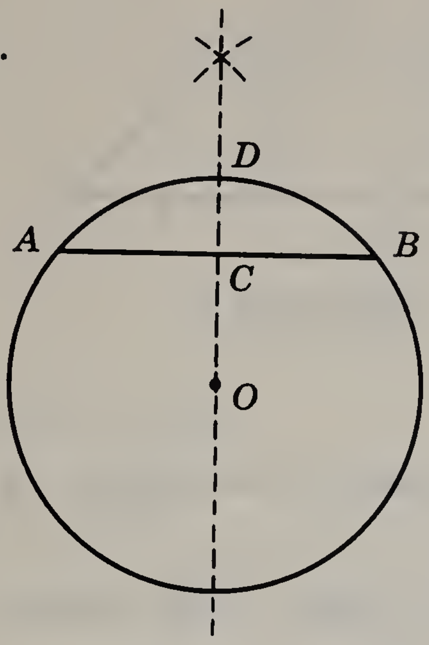
TEST 13 (CHAPTER 11)

Part 1. Multiple-Choice Statements

1. c. 2. d. 3. d. 4. a. 5. d. 6. b. 7. c.
8. d. 9. a. 10. b.

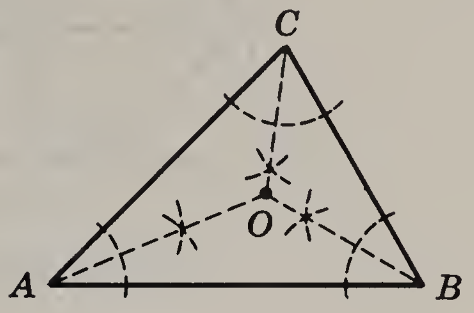
Part 2. Constructions

11.



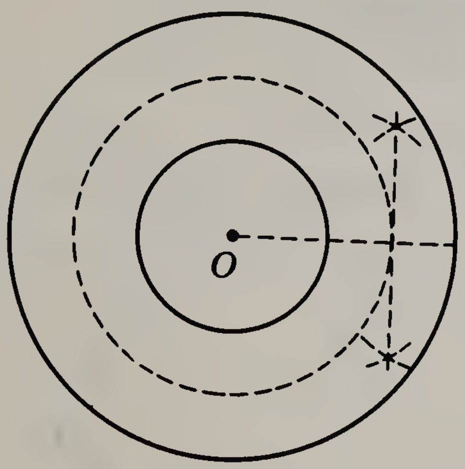
The locus is CD

12.



The required locus is the point O.

13.



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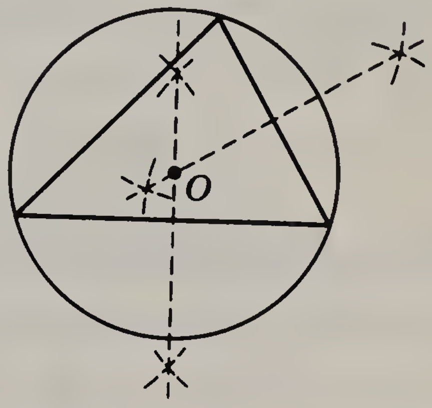
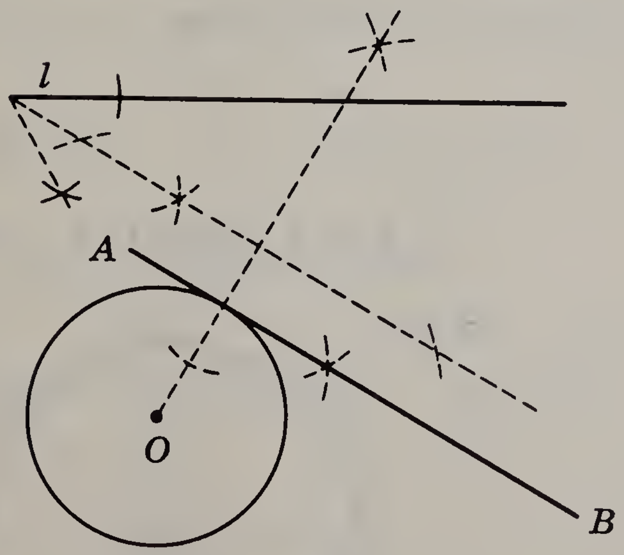


Point O is the circumcenter of the △.

15.



AB is the required line.

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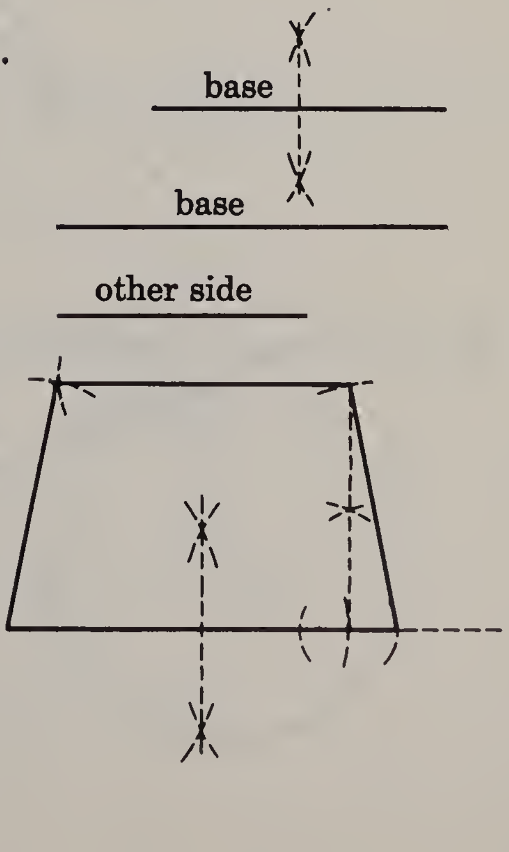

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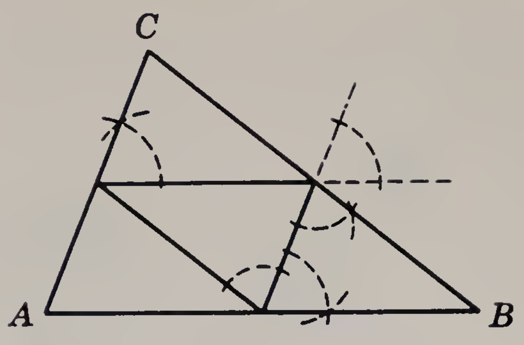
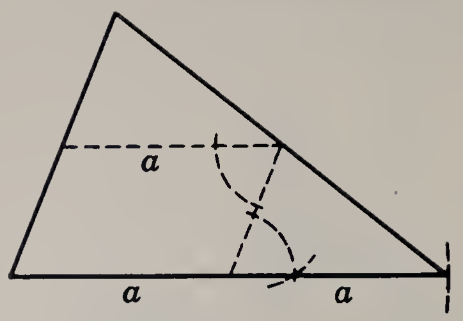


Solution 1



Solution 2

Part 3. Miscellaneous

18. Any point equidistant from the sides of an angle is on the bisector of the angle.

19. Any point is equidistant from the sides of an angle, if and only if, it is on the bisector of the angle.

20. The necessary and sufficient condition for a point to be on the bisector of an angle is that it be equidistant from the sides of the angle.

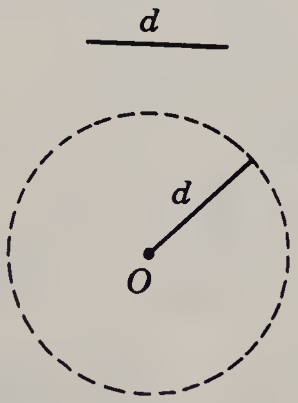
TEST 14 (CUMULATIVE. CHAPTERS 9-11)

Part 1. Multiple-Choice Statements

1. c. 2. c. 3. d. 4. a. 5. a. 6. d. 7. d. 8. b. 9. b. 10. d.

Part 2. Constructions

11.



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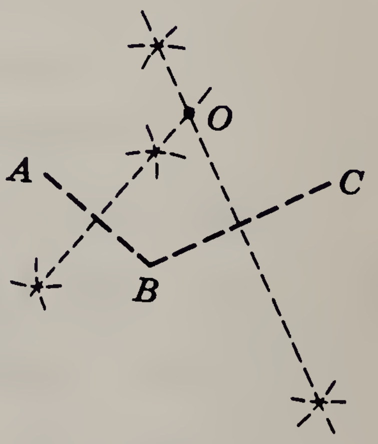
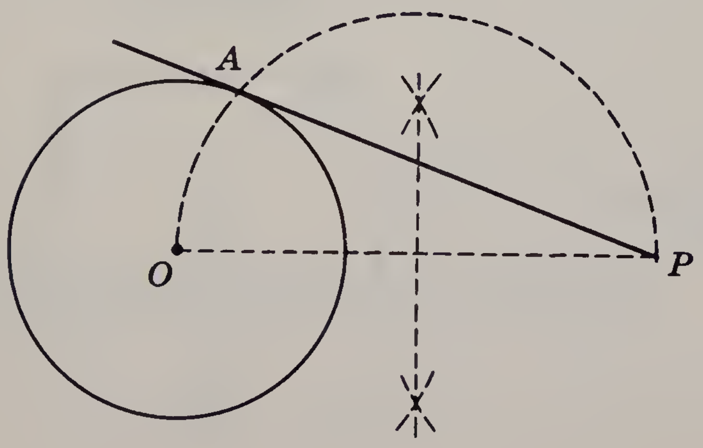


O is the required locus.

13.



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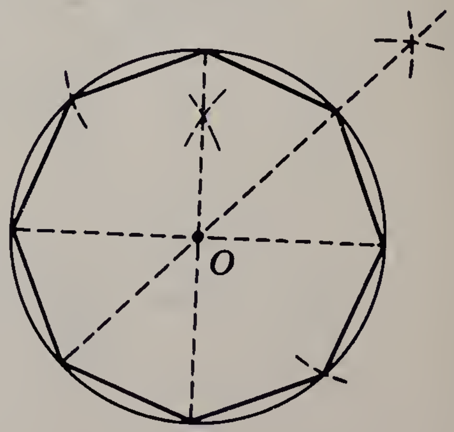

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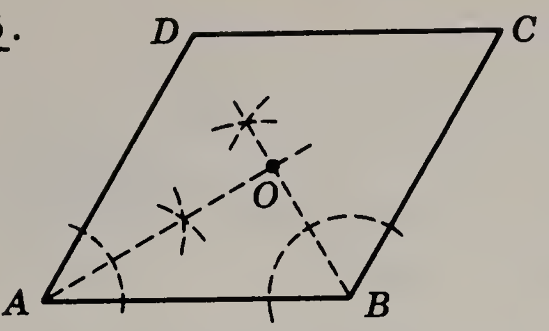


O is the required locus.

Part 3. Proofs

16. Proof: 1. AB is tangent to ⊙ O. (Given.) 2. Chord PM = chord PN. (Given.) 3. $\widehat{PM} = \widehat{PN}$. (§286.) 4. $\frac{1}{2}\widehat{PM} = \frac{1}{2}\widehat{PN}$. (Asmt. 4. 5. $\angle APM \overset{\circ}{=} \frac{1}{2}\widehat{PM}$. (§322.) 6. $\angle BPN \overset{\circ}{=} \frac{1}{2}\widehat{PN}$. (§322.) 7. $\angle APM = \angle BPN$. (Asmt. 7.)

17. Proof: 1. Draw BD. (Asmt. 11.) 2. ADC is a straight line and AD = DC. (Given.) 3. AB is a diameter of ⊙ O. (Given.) 4. $\angle ADB$ is inscribed in a semicircle. (§§279,283.) 5. $\angle ADB$ is a rt. $\angle$. (§319.) 6. BD $\perp$ AC. (§14.) 7. From (6) and (2), AB = BC. (§79.) 8. Then $\triangle$ ABC is isosceles. (§56.)

18. Proof: 1. ⊙ O and ⊙ O' are tangent externally at E. (Given 2. CD passes through E. (Given.) 3. AB passes through E. (§297.) 4. $\widehat{ACE}$ and $\widehat{EDB}$ are semicircles. (§283.) 5. $\angle C$ and $\angle D$ are rt. $\angle$s. (§319.) 6. AC $\perp$ CD and BD $\perp$ CD. (§14.) 7. AC $\parallel$ BD. (§107.)

TEST 15 (CHAPTER 12)

Part 1. Completing Statements

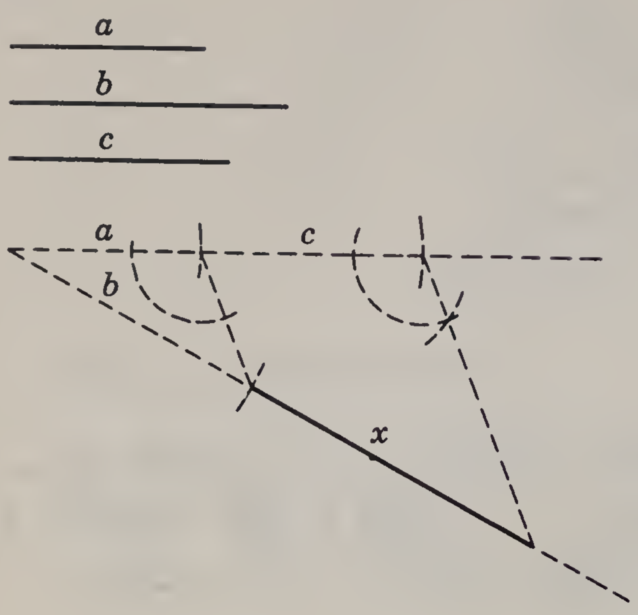
1. proportion. 2. means. 3. proportionally. 4. adjacent. 5. harmonically.

Part 2. Applications

6. 20. 7. ±20. 8. 1. 9. 9 inches and 15 inches. 10. 4. 11. $2\frac{1}{2}$. 12. 10. 13. $13\frac{1}{3}$. 14. 18.

Part 3. Constructions

15.



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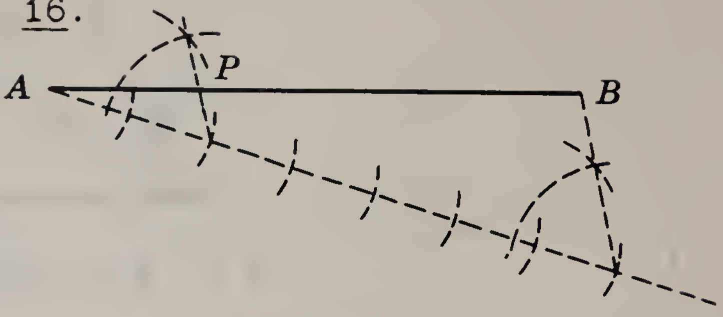
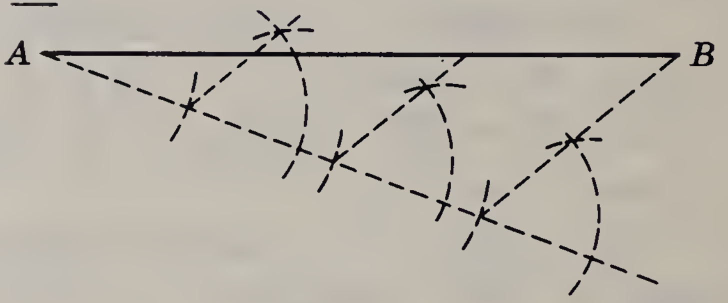


Point P divides AB into
ratio 2 : 5.

17.



Part 4. Supplying Reasons

18. §386. 19. §392. 20. §393. 21. §388. 22. §387. 23. §403. 24. §402.

Part 5. Proofs

25. Proof: 1. ABCD is a trapezoid with bases AB and CD. (Given.) 2. AB ∥ CD. (§206.) 3. BD is a st. line. (§182.) 4. $\angle DCO = \angle BAC$. (§111.)

26. Proof: 1. In $\triangle ABC$, FE ∥ AB and DE ∥ AC. (Given.) 2. Then $\frac{CF}{FA} = \frac{CE}{EB}$ and $\frac{CE}{EB} = \frac{AD}{DB}$. (§397.) 3. Then $\frac{CF}{FA} = \frac{AD}{DB}$. (Asmt. 7.)

TEST 16 (CHAPTER 13)

Part 1. True-False Statements

1. True. 2. False. 3. True. 4. True. 5. False. 6. True. 7. False. 8. False. 9. False. 10. True.

Part 2. Applications

11. 27. 12. 6. 13. 18. 14. 4 : 9. 15. $4\sqrt{5}$ in. and $2\sqrt{5}$ in. 16. $2\frac{1}{2}$ in. 17. $2\sqrt{13}$ in. and $3\sqrt{13}$ in. 18. 6 in. and 8 in. 19. 20 in. 20. 12 in.

Part 3. Supplying Reasons

21. Given. 22. §295. 23. §14. 24. §57. 25. §107. 26. §111. 27. §318. 28. §322. 29. Asmt. 7. 30. Asmt. 6. 31. §416. 32. §412.

Part 4. Proofs

33. Proof: 1. AB is a diameter of $\odot$ O. (Given.) 2. BC is tangent to $\odot$ O at B. (Given.) 3. BC $\perp$ AB. (§295.) 4. $\angle$ ABC is a rt. $\angle$. (§14.) 5. $\angle$ AEB is inscribed in a semicircle. (§283.) 6. $\angle$ AEB is a rt. $\angle$. (§319.) 7. $\angle$ ABC = $\angle$ AEB. (Asmt. 19.) 8. $\angle$ A = $\angle$ A. (Iden.) 9. $\triangle$ ABE $\sim$ $\triangle$ ABC. (§415.) 10. $\frac{AC}{AB} = \frac{AB}{AE}$. (§412.)

34. Proof: 1. In $\triangle$ABC, AC = BC. (Given.) 2. $\angle$A = $\angle$ABC. (§83.) 3. In $\triangle$ABP, AB = PB. (Given.) 4. Then $\angle$A = $\angle$APB. (§83.) 5. Then $\angle$ABC = $\angle$APB. (Asmt. 7.) 6. $\angle$A = $\angle$A. (Iden.) 7. $\triangle$ABP $\sim$ $\triangle$ ABC. (§415.) 8. $\frac{AP}{AB} = \frac{AB}{AC}$. (§412.)

35. Proof: 1. CD is a common external tangent of $\odot$ O and $\odot$ O'. (Given.)

2. P is the intersection of the common secant PA and CD. (Given.)

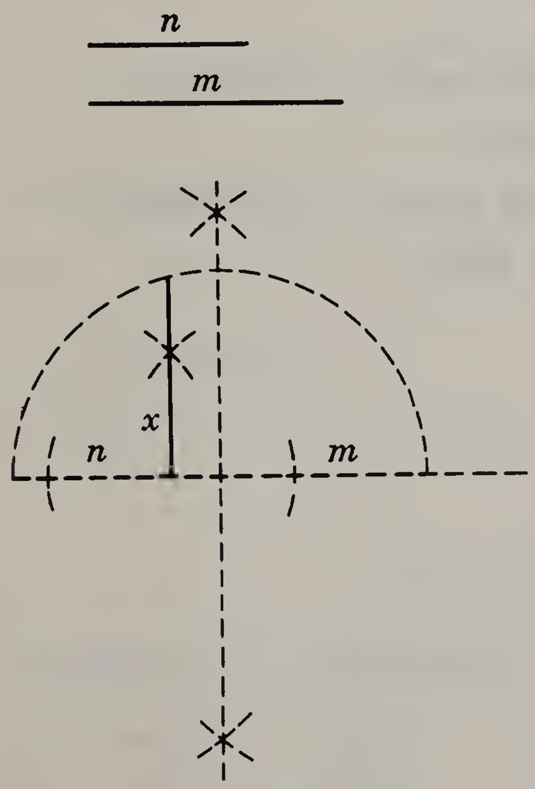
3. $\frac{PB}{PD} = \frac{PD}{PA}$ and $\frac{PB}{PC} = \frac{PC}{PA}$. (§433.)

4. $\overline{PD}^2 = PB \times PA$ and $\overline{PC}^2 = PB \times PA$. (§386.)

5. $\overline{PD}^2 = \overline{PC}^2$. (Asmt. 7.) 6. PD = PC. (Asmt. 8.)

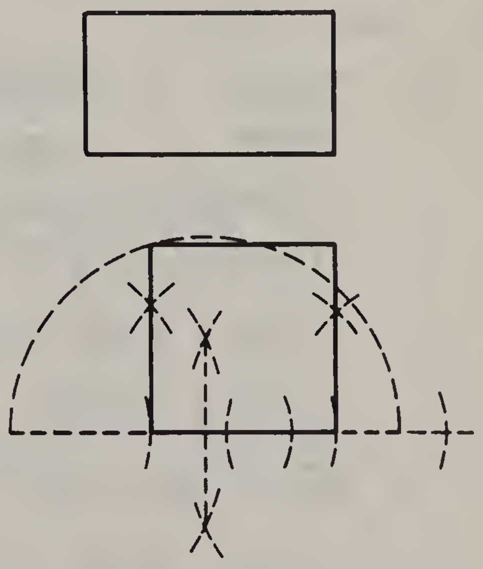
Part 5. Constructions

36.



x is the mean proportional between m and n

37.



TEST 17 (CHAPTER 14)

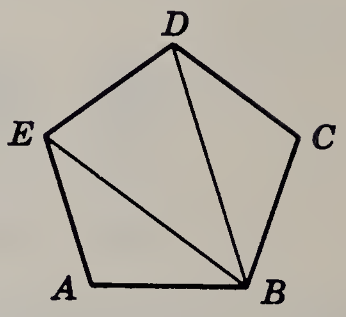
Part 1. Completing Statements

1. $\frac{360}{n}$. 2. $s = \frac{1}{2}ap$. 3. radius. 4. areas. 5. regular.
6. segment. 7. k. 8. r^2. 9. tangents. 10. 4.

Part 2. Applications

11. $51\frac{3}{7}$. 12. 144. 13. $54\sqrt{3}$ sq.in. 14. 45°.
15. $2\sqrt{3}$ in. 16. 10 in. 17. 2 : 3. 18. 24 in.
19. $\frac{27\pi}{8}$ sq.in. 20. 4 : 5. 21. 196 sq.in. 22. 8 in.
23. 113.04 sq.in. 24. 28π sq.in., or 87.92 sq.in.
25. $(4\sqrt{5} - 4)$ in., or 4.94 in.

Part 3. Proofs



26. Given BE and BD any two diagonals of regular pentagon ABCDE.

To prove that BE = BD.

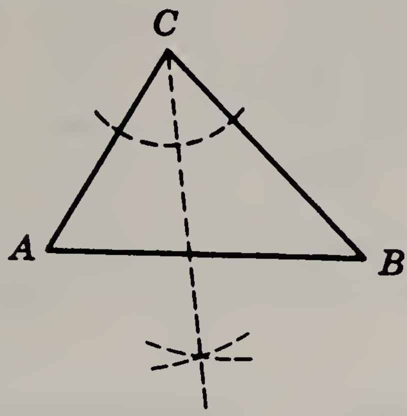
Proof: 1. ABCDE is a regular pentagon. (Given.) 2. ABCDE is a regular polygon. (§183.) 3. AE = BC and AB = CD. (§184.) 4. $\angle A = \angle C$. (§184.) 5. $\triangle ABE \cong \triangle BCD$. (Asmt. 32.) 6. BE = BD. (Asmt. 34.)

27. Proof: 1. ABCDE is a regular pentagon. (Given.)
2. ABCDE is a regular polygon. (§183.)
3. Diagonals AD and BE intersect in point P. (Given.)
4. Circumscribe $\odot O$ about ABCDE. (§458.)
5. $\angle DPE \overset{\circ}{=} \frac{1}{2}(\widehat{AB} + \widehat{ED})$. (§323.)
6. $\angle PED \overset{\circ}{=} \frac{1}{2}(\widehat{BCD})$. (§318.)
7. $\widehat{BCD} = \widehat{BC} + \widehat{CD}$. (Asmt. 9.)
8. $\angle PED \overset{\circ}{=} \frac{1}{2}(\widehat{BC} + \widehat{CD})$. (Asmt. 6.)
9. BC = AB and CD = ED. (§184.)
10. $\widehat{BC} = \widehat{AB}$ and $\widehat{CD} = \widehat{ED}$. (§286.)
11. $\angle PED \overset{\circ}{=} \frac{1}{2}(\widehat{AB} + \widehat{ED})$. (Asmt. 6.)
12. $\angle PED = \angle DPE$. (Asmt. 7.)
13. PD = ED. (§128.)

TEST 18 (CUMULATIVE. CHAPTERS 12-14)

1. $6\frac{2}{3}$. 2. $8\frac{1}{3}$. 3. $\frac{AX}{AC} = \frac{AC}{AB}$. 4. $\frac{AX}{CX} = \frac{CX}{XB}$. 5. 15. 6. $\frac{POX}{P'O'X} = \frac{4}{9}$. 7. 6. 8. $320. 9. 9. 10. △ABC and △ADB. 11. 100π. 12. 16π.

13.



14. Since PE is a secant to each circle, $\overline{PA}^2 = PF \times PE$, $\overline{PB}^2 = PF \times PE$, $\overline{PC}^2 = PF \times PE$, and $\overline{PD}^2 = PF \times PE$. (§433.) Then $\overline{PA}^2 = \overline{PB}^2 = \overline{PC}^2 = \overline{PD}^2$ (Asmt. 7.) and PA = PB = PC = PD. (Asmt. 8.) Then a circle with P as a center and a radius = PA will pass through A, B, C, and D. (Asmt. 47.)

15. Proof: 1. DE bisects AB and CB of △ ACB. (Given.) 2. DE ∥ AC. (§204.) 3. ∠BDE = ∠A and ∠DEB = ∠C. (§112.) 4. △ DEB ∼ △ACB. (§415.)

TEST 19 (CHAPTER 15)

Part 1

1. opposite leg. 2. the hypotenuse. 3. cos x. 4. cos A. 5. 1.

Part 2. Applications

6. .385. 7. .417. 8. .3692. 9. 54°27'. 10. 6.12 ins.

TEST 20 (CHAPTER 16)

Part 1. Completing Statements

1. the reverse order. 2. the same order. 3. perpendicular. 4. nearer. 5. greater. 6. less than. 7. decreased. 8. the greater.

Part 2. Supplying Reasons

9. Asmt. 53. 10. Asmt. 54. 11. §521. 12. §520.

Part 3. Proofs

13. Proof: 1. CE is a median of $\triangle ABC$. (Given.) 2. AE = EB. (§89.) 3. AC = CE. (Given.) 4. $\angle A = \angle CEA$. (§83.) 5. $\angle CEB = \angle A + \angle ACE$. (§123.) 6. $\angle CEB = \angle CEA + \angle ACE$. (Asmt. 6.) 7. $\angle CEA + \angle ACE > \angle CEA$. (Asmt. 9.) 8. $\angle CEB > \angle CEA$. (Asmt. 6.) 9. CE = CE. (Iden.) 10. From (2), (9), and (8), BC $>$ AC. (§524.) 11. Then BC $>$ CE. (Asmt. 6.)

14. Proof: 1. PAO is a st. line drawn from external point P to O of $\odot$ O. (Given.) 2. PB + BO $>$ PO. (§519.) 3. PO = PA + AO. (Asmt. 9.) 4. PB + BO $>$ PA + AO. (Asmt. 6.) 5. BO = AO. (Asmt. 26.) 6. PB $>$ PA, or PA $<$ PB. (Asmt. 54.)

TEST 21 (CHAPTER 17)

Part 1. Applications

1. $\sqrt{130}$. 2. (1, 1). 3. $-2\frac{1}{4}$. 4. $x + 2y - 8 = 0$. 5. $5x - 3y + 15 = 0$. 6. $x^2 - 10x + y^2 - 24y = 0$. 7. (4, 3). 8. $x - y = 0$. 9. The center is (-1, 2) and the radius is 5. 10. 1.

Part 2. Proofs

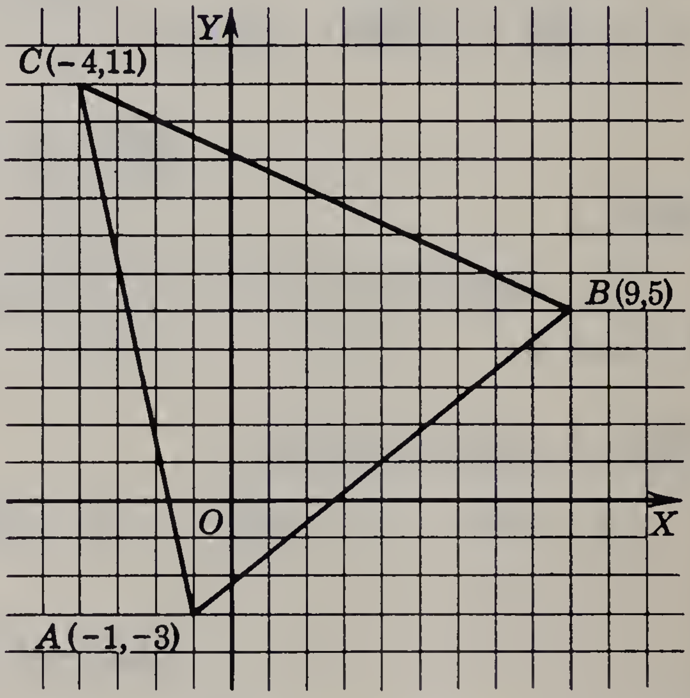
11. Proof:

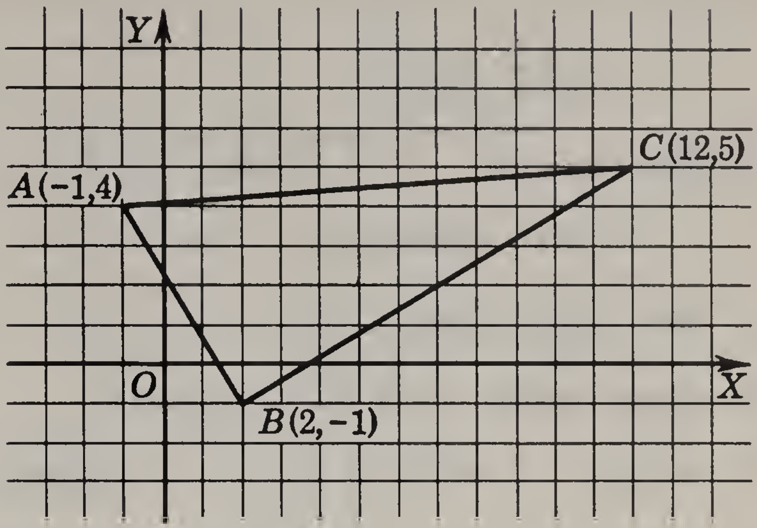
1. $AB = \sqrt{(9 + 1)^2 + (5 + 3)^2}$
   $= \sqrt{164}$ (§540)

2. $AC = \sqrt{(-4 + 1)^2 + (11 + 3)^2}$
   $= \sqrt{205}$ (§540)

3. $BC = \sqrt{(-4 - 9)^2 + (11 - 5)^2}$
   $= \sqrt{205}$

4. Since AC = BC, $\triangle ABC$ is isos. (§56.)





12. Proof:

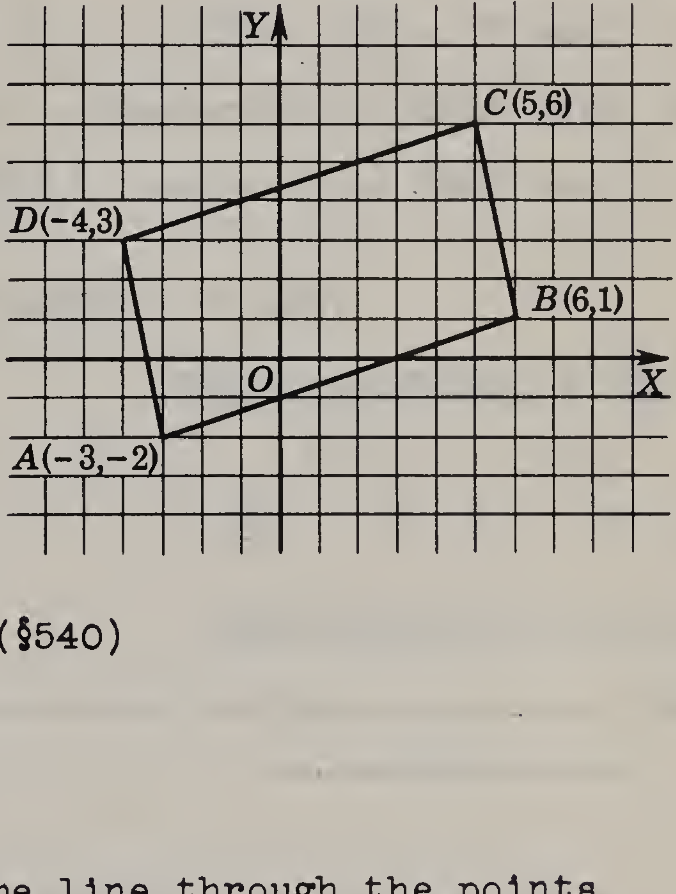
1. The slope of AB $= \frac{4 + 1}{-1 - 2}$
$= \frac{5}{-3}$ (§537)

2. The slope of BC $= \frac{5 + 1}{12 - 2}$
$= \frac{3}{5}$ (§537)

3. $\frac{5}{-3} \times \frac{3}{5} = -1$

4. Then $AB \perp BC$. (§538.)

5. ∠B is a rt. ∠. (§14.)   6. △ABC is a rt. △. (§57.)



13. Proof:

1. $AB = \sqrt{(6 + 3)^2 + (1 + 2)^2}$
$= \sqrt{90}$ (§540)

2. $DC = \sqrt{(5 + 4)^2 + (6 - 3)^2}$
$= \sqrt{90}$ (§540)

3. Then AB = DC. (Asmt. 7.)

4. $AD = \sqrt{(-4 + 3)^2 + (3 + 2)^2}$
$= \sqrt{26}$ (§540)

5. $BC = \sqrt{(5 - 6)^2 + (6 - 1)^2} = \sqrt{26}$ (§540)

6. Then AD = BC. (Asmt. 7)

7. Then ABCD is a ▱. (§197)

14. Proof: 1. The equation of the line through the points (3,1) and (0,-1) is $\frac{y - 1}{x - 3} = \frac{-1 - 1}{0 - 3}$, or $2x - 3y = 3$. (§546.)

2. If $x = -6$, and $y = -5$, $-12 + 15 = 3$. Then the point (-6,-5) is on the line passing through (3,1) and (0,-1).

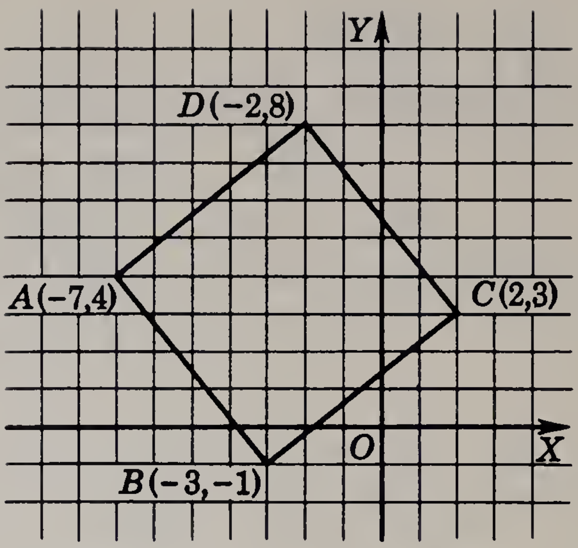
Note. This Exercise can be proved by showing that the line through the points (3,1) and (0,-1) has the same slope as the line through the points (0,-1) and (-6,-5).

15. Proof:

1. $BC = \sqrt{(2 + 3)^2 + (3 + 1)^2} = \sqrt{41}$ (§540)

2. $AD = \sqrt{(-2 + 7)^2 + (8 - 4)^2} = \sqrt{41}$ (§540)

3. Then BC = AD. (Asmt. 7)



4. $AB = \sqrt{(-7 + 3)^2 + (4 + 1)^2}$

   $= \sqrt{41}$ (§540)

5. $CD = \sqrt{(-2 - 2)^2 + (8 - 3)^2}$

   $= \sqrt{41}$ (§540)

6. AB = CD. (Asmt. 7.)

7. Then ABCD is a ▱. (§197)

8. Slope of BC = $\frac{3 + 1}{2 + 3} = \frac{4}{5}$ (§536)

9. Slope of CD = $\frac{8 - 3}{-2 - 2} = \frac{5}{-4}$ (§536)

10. $\frac{4}{5} \times \frac{5}{-4} = -1$

11. Then BC ⊥ CD. (§538.) 12. Then ∠C is a rt. ∠. (§14.)

13. Then ABCD is a rectangle. (§185.) 14. BC = CD. (Asmt. 7.)

15. Then ABCD is a square. (§185.)

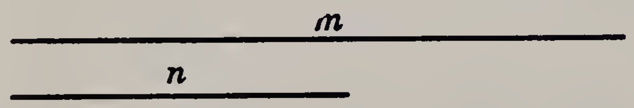
TEST 22 (CUMULATIVE. CHAPTERS 1-17)

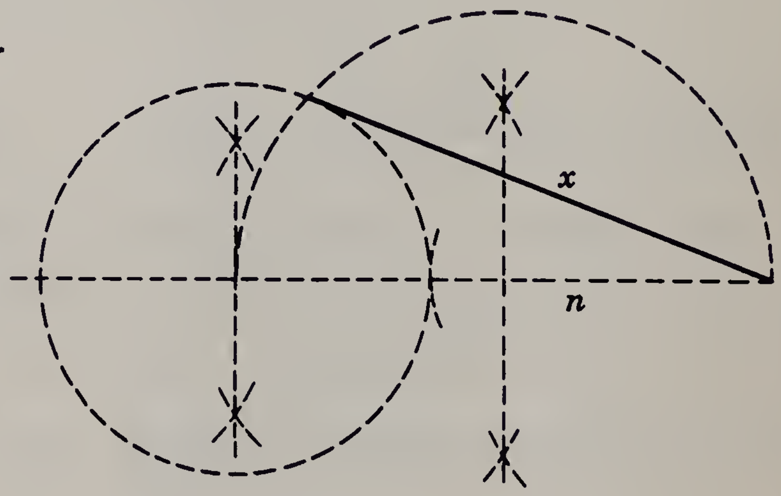
Part 1. Multiple-Choice

1. b. 2. d. 3. b. 4. b. 5. a. 6. d. 7. c.
8. d. 9. c. 10. b.

Part 2. Constructions

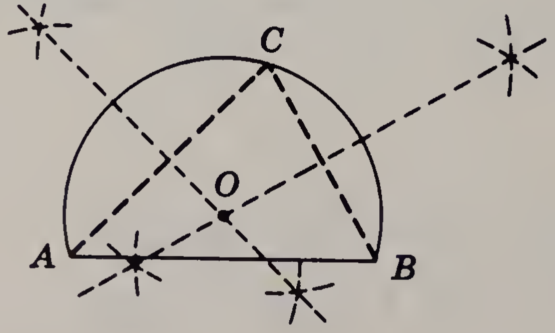
11.





x is the mean proportional between m and n

12.



$\overset{\frown}{ACB}$ is the required locus.

13. Proof: 1. DE is the ⊥ bisector of BC. (Given.) 2. DC = DB. (§79.) 3. ∠x = ∠B. (§ 83.) 4. AC = BC. (Given.) 5. ∠A = ∠B. (§83.) 6. ∠A = ∠x. (Asmt. 7.)

14. Proof: 1. PA is tangent to ⊙O. (Given.) 2. PA ⊥ radius OA. (§295.) 3. ∠OAP is a rt. ∠. (§14.) 4. △OAP is a rt. △. (§57.) 5. AB ⊥ OP. (Given.) 6. BP is the projection of AP on OP. (§426.) 7. $\frac{BP}{AP} = \frac{AP}{OP}$. (§ 427.) 8. $\overline{AP}^2 = OP \times BP$. (§386.)

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